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Studies in Bhartrhari's Vakyapadiya.

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STUDIES IN BHARTṚHARI'S VĀKYAPADĪYA

A Dissertation Presented

by

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ABSTRACT

Studies in Bhartṛhari's Vākyapadīya

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Bhartṛhari's Vākyapadīya was the work in which the speculations of the Indian Grammarians found their fullest expression. In this dissertation three topics treated in the Vākyapadīya are explicated: ākāṅkṣā, sphoṭa, and śabdabrahman. A Montague-style categorial grammar for a fragment of Sanskrit is given to provide an explication of the term ākāṅkṣā. The semantics of that fragment is discussed in connection with the concept sphoṭa and it is argued that Bhartrhari's theory of sphoṭa was not the one currently attributed to him by scholars of his work. The concept of śabdabrahman is discussed with reference to two questions: Why did Bhartrhari think that the study of Grammar leads to salvation, and how can a monistic position be consistently asserted? The formal machinery of the earlier chapters is employed in these discussions. Another concept--that of metalanguage hierarchies--is discussed but the intent is to demonstrate that such hierarchies need not be employed in a formal treatment of Bhartrhari's work and a different system is given within which such hierarchies may be accommodated without recourse to a different language.

TABLE OF CONTENTS

I N T R O D U C T I O N

This dissertation is a Western treatment of three concepts discussed in the Vākyapadīya of the Indian philosopher of language Bhartṛhari. The following remarks may be useful as background information to the reader.

1. Concerning The Grammarians[1]

According to tradition the first Indian Grammarian was the god Indra, who received instruction from the god Bṛhaspati[2]. Many analyses of the syntax of Sanskrit were proposed in the years which followed this event, culminating in Pāṇini's Aṣṭādhyāyī, written (most likely) in the fifth century b.c.. Pāṇini's stature among Western linguists is great, as these remarks by Leonard Bloomfield indicate:

> (1) [Pāṇini's grammar is] one of the greatest monuments of human intelligence....It describes, with the minutest detail, every inflection, derivation, and composition, and every syntactic usage of its author's speech. No other language, to this day, has been so perfectly described. (Language, 1933, New York, p. 11)
>
> (2) Hindu grammar described the Sanskrit language completely and in scientific terms, without prepossessions or philosophical intrusions. It was from this model that Western scholars learned, in the course of a few decades, to describe a language in terms of its own structure. ("Linguistic Aspects of Science", Bloomfield (1939))

Pāṇini's stature in India was greater still; he is frequently referred to as "divine" in the literature and tradition has it that some verses of his Aṣṭādhyāyī were divinely dictated.

Pāṇini's treatment of Sanskrit syntax had two effects. It elevated the version of Sanskrit which he formalized to the level of

a divine language, rendering it immune to the ordinary processes of linguistic change, and it provided a solid basis for speculations of the Indian schools of philosophy about the nature of language and meaning.

The Grammarians of India themselves formed such a school, called the Vyākaraṇas. (In this work I use the English term "Grammarians" for them.) After Pāṇini came several Grammarians whose work has been lost, including Vyādi and Kātyāyana. They were followed by Patañjali, who wrote a famous commentary on Pāṇini's Aṣṭādhyāyī, the Mahābhāṣya. Other Grammarians succeeded Patañjali in the period up to the fifth century a.d., but little is known of them, except that the Grammarian school had dwindled in numbers and many of its adherents were not capable of understanding Pāṇini's work directly. At that time a Grammarian named Bhartṛhari, having been instructed by a guru named Vasurāta, wrote a commentary on Patañjali's Mahābhāṣya and a book of his own, the Vākyapadīya, which set forth the theories of language which were known in Bhartṛhari's time, contrasting them with his own views. This book is regarded by many scholars as the highest point in the history of Indian thought about the nature of language.

There were other Grammarians who came after Bhartṛhari. Two of them--Bhaṭṭoji Dīkṣita and Nāgeśa Bhaṭṭa--expanded Bhartṛhari's theory of sphoṭa and seem to have modified it. The Grammarian school continued to grow and develop in India. At the present time it has a number of adherents and the educated Indian learns Sanskrit by working with Pāṇini's grammar.

2. Concerning Bhartṛhari

About Bhartṛhari's life there is little known for certain. The most extensive historical account of him is that of the Chinese pilgrim I-tsing, a Buddhist who made a journey to India in the seventh century a.d..[3] I-tsing wrote that a Grammarian named Bhartṛhari had died about 650 a.d., had been a Buddhist, had vacillated between the secular and holy ways of life seven times, had written a commentary on the Mahābhāṣya of Patañjali, had written a work called the Vākya-discourse in two books with a commentary, and had written another volume called Peina.

Much of this information--in particular, all of it relating to Bhartṛhari's private life--has been shown to be doubtful. Iyer writes that "There is enough evidence to push [Bhartṛhari's] date back by at least two centuries."[4] That Bhartṛhari was a Buddhist seems doubtful, given his reliance on the Vedas and his adherence to the Advaita Vedānta interpretation of them. His vacillation between secular and holy life is generally attributed to the poet Bhartṛhari, who wrote a series of verses called the Subhāṣitatriśatī. There is a good deal of dispute whether the Grammarian Bhartṛhari was the poet Bhartṛhari, and the best informed opinion today seems to be that we do not know who wrote the poems of Bhartṛhari.[5] Fragments of Bhartṛhari's commentary on the Mahābhāṣya exist today, as do the two works cited by I-tsing. Taken together, the Vākya-discourse and the Peina make up the Vākyapadīya, the subject of this dissertation.

What we know more or less for certain of Bhartṛhari, then, is this: He lived in the fifth century a.d., was the pupil of a

Grammarian named Vasurāta, wrote a commentary on Patañjali's work, and wrote the treatise in which the speculation of the Grammarians culminates. The rest is lost in the fifteen centuries which come between his time and our own.

3. Concerning the Vākyapadīya

I assume that the Vākyapadīya is composed of three books, although there is some dispute on this point. Pillai holds that only Books I and II are properly called the Vākyapadīya, citing as evidence the facts that Bhartṛhari's and Puṇyarāja's commentaries only cover Books I and II, I-tsing's account of the size of the Vākya-discourse is that of Books I and II, Helārāja's commentary is on Book III alone, and the conclusion of Book II is stylistically the termination of a work rather than a less-than-final part of it.[6] Pillai's translation, entitled The Vākyapadīya, is of BookI and Book II alone.

Iyer disputes this account of the composition of the Vākyapadīya,[7] citing as strongest evidence that all three books constitute it the following verse from Book II:

> (3) Here (i.e., in Cantos I and II) only the bare essentials of a few systems are given. There will be a study in details in Canto III. (VP II 483, Pillai)

Pillai does not discuss this verse when he cites evidence for his views on the composition of the Vākyapadīya, perhaps viewing it as a later interpolation.

Concerning these matters my intention has been to provide an account of certain concepts of Bhartṛhari's philosophy of language. There is no dispute concerning the authorship of the three books;

the views in those three books cohere; passages from all three illuminate the theories contained in any one of them. For these reasons I treat them as a unit and call the group of the three of them the Vākyapadīya. One who is sympathetic to Pillai's views may read the phrase "the Vākyapadīya" as "the Vākyapadīya and the Prakīrṇaka" wherever it appears in the text.

There are some disputed interpretations of texts and a few apocryphal verses. The apocryphal verses aren't crucial to the research presented here, but in several cases the disputed texts are crucial. In such cases the fact is noted.

It is difficult to characterize the subject matter of the three books of the Vākyapadīya in an ordered way because each discusses a wide variety of topics, often without transition, and points made in one book are often brought up or elaborated in another. Traditionally the first book is known as the Brahmakāṇḍa (Brahman-book), the second as the Vākyakaṇḍa (Sentence-book), and the third as the Padakāṇḍa (Word-book). However the most useful passages about Brahman are found in Book III and a good deal of discussion of the nature of words is found in Book II. For these reasons I do not attempt a summary of the ideas presented in each book, relying instead on quotation to present the ideas in the texts as I discuss them.

I have used Pillai's translation and numbering of the verses of Books I and II for purposes of quotation and have had recourse to Biardeau's translation into French of Book I and its Commentary when puzzled, although I do not quote from it here. For Book III I have quoted from Iyer's translation. Whenever a passage is quoted from the Vakyapadiya, the letters "VP" follow it with the name of

the translator and the passage number. The Sanskrit originals are not quoted, because this dissertation is written for Western readers, and the Sanskrit original of the translated passages would occupy a good deal of space without enlightening the intended reader.

4. Concerning The Commentators

The following commentators are referred to at many points in the chapters to come: Pillai for his remarks on Books I and II which precede the text, for his transitions between verses, and for his parenthesized passages interspersed with the translations which make clear the sense of a passage; Iyer for his transitions between verses of Book III, for his elaboration of difficult verses, for his summary of Helārāja's commentary, and for Iyer(1969), a volume devoted to Bhartṛhari's work upon which I have relied heavily; and Helārāja's commentary on Book III, written early in the tenth century a.d. and summararized in Iyer's translation of Book III.

There are other ancient commentators on the Vākyapadīya who have influenced Pillai and Iyer in their translations, but who have not had direct influence on the research which follows, save the Vṛtti of Harivṛṣabha (which may be a commentary of Bhartṛhari's) in Biardeau's translation of Book I.

There are modern scholars of the Grammarian school who are quoted frequently in the chapters to come: Brough, Kunjunni Raja (sometimes referred to as "Raja" in the text), Chakravarti, and Staal.

The Vākyapadīya is written in verse form so that it will be easier to memorize. For this reason it contains highly compressed treatments

of many of the topics discussed in it. It is hard going and without the aid of the commentators much of what is said is lost to the reader. I have relied to a great extent on the elaborations and interpretations of the commentators in the chapters which follow, departing from their views only when it seems philosophically expedient to do so.

5. Concerning this Work

My training has been primarily in the area of Western philosophy, with particular attention paid to the philosophy of language. I first read about the Grammarians to satisfy a point I was curious about. My interest was piqued and it deepened until it has become something of an obsession, culminating in the plan to write a dissertation which presented a unified account of Bhartṛhari's thought. Problems of time and space have eroded that plan so that it is only three concepts of Bhartṛhari's theory which are thoroughly discussed in the chapters which follow--the notions of ākāṅkṣā, sphoṭa, and śabdabrahman. These three concepts are connected in Bhartṛhari's thought and stand at the heart of it, but there is much in what he wrote which is not touched on in an exposition of them.

Some characterization of these three concepts may serve to orient the reader in the discussions to follow. Ākāṅkṣā, literally, 'expectancy', is a syntactic notion used to describe the following feature of words: words or phrases of different sorts have expectancy for words or phrases of other sort. If such expectancy is satisfied a sentence results. This concept was not originated by the Grammarians,

having developed in the Mīmāṃsā philosophy of language. Its application is based on the syntactic work of Pāṇini, however, and Bhartṛhari uses it in his discussions of the nature of the sentence as a sequence of words. For these reasons I take some pains to formalize the notion of ākāṅkṣā by means of a categorial grammar of the Montague sort for a fragment of a version of Sanskrit. (The version, it should be noted, treats words taken out of sandhi for simplicity. This feature of it will shock the experienced reader of Sanskrit, although the student of Perry's A Sanskrit Primer will recognize the fragment as that of the early lessons in Perry's book.) The fragment has quantifiers, intensional contexts, and translations into the predicate calculus--all the most recent features of the Montague style of doing semantics. The grammar and its informal presentation take up Chapter I and Chapter II of this dissertation.

Sphoṭa is a concept which is peculiar to the Grammarian school. Kunjunni Raja writes,

> (4) Next to the Sphoṭa theory of linguistic symbols advocated by Bhartṛhari...the most important contribution of ancient India to general linguistics is the concept of ākāṅkṣā.

Some commentators are not so kind to the doctrine; I believe that this fact is related to the fact that it seems to have been misinterpreted. The doctrine itself has to do with language viewed as an integral unit. Crudely put, Bhartṛhari would say that words express a sentence-sphoṭa, which is an indivisible entity. (In particular, sentence-sphoṭas are not divided into words or word-meanings). This view was in contrast to the view of the Mīmāṃsakas, which was that sentences are

conglomerations of words and sentence-meanings conglomerations of word-meanings. It was the doctrine of sphoṭa which first interested me in Bhartṛhari; partly because it seemed lovely, dark and deep; partly because I sensed an obstruction between the theory and me, the result of misinterpretation by Bhartṛhari's successors and commentators. Two years later I believe I have identified the obstruction, explained why it is there, and redeveloped the sphoṭa theory which Bhartṛhari actually held. (The reader will have to decide this point for his or her self.) In treating the doctrine of sphoṭa some changes are rung on the Sanskrit fragment introduced earlier. The intent is to provide a formal version of the idea that meanings may be assigned to letters in such a way that sentence-meanings are derived from the interaction of letter-meanings. These topics occupy Chapter III of this dissertation.

Chapter IV is devoted to a demonstration that the machinery of metalanguages usually employed in discussions of semantics need not (indeed, ought not) be used in representing Bhartṛhari's thought. In a sense this chapter constitutes a study of a fourth concept in Bhartṛhari's work, that of the metalanguage. Since the object of the study is to prove that Bhartṛhari did not use the concept and need not have used it, I have not counted it as a concept which is explicated here. In fact, my intention is to bring about the whole-scale demolition of the metalanguage idea as a sine qua non of semantic discourse.

Chapter V treats Bhartṛhari's theory of Brahman (the ultimate reality of the Vedic tradition) as it is related to his theory of language. It may seem peculiar that this doctrine should figure in a

work devoted to a careful treatment of syntactic and semantic concepts. This apparent anomaly deserves discussion.

In order to understand the Vākyapadīya it is necessary to understand its connection to the Vedic tradition, for it was written to defend that tradition and at times it invokes the Vedas in its own defense. Bhartṛhari's belief was the Vedantic belief that the Vedas are eternal documents whose function is to show people the way to salvation--union with Brahman. The latter sections of the Vedas, the Upaniṣads, were regarded by Vedāntists as the summation of Indian religious thought in terms of statements about Brahman. It was common for Brahman to be characterized in the Vedas as linguistic in nature, as in this passage:

(5) The highest Brahma, your Majesty, is in truth speech.
(Bṛhad-Āraṇyaka Upaniṣad, IV.1.2, Hume p. 127)

In this tradition lies part of the explanation for Bhartṛhari's preoccupation with the relation between grammar and Brahman--a relation which is referred to frequently in the course of the Vākyapadīya. Another part of the explanation lies in the tradition that Grammar is a divine discipline. Following the traditional account, the first Grammarian--Indra--was divinely taught and this beginning seems to have lent to linguistics the status of the primary science in India. (It is colloquially known as the "Veda of the Vedas"[8]). This is in comparison to the Western view that mathematics is the queen of the sciences. Speaking very generally, this difference in stress may account for the fact that God is characterized as the ultimate geometer or the ultimate mathematical entity in the West and that Brahman is

characterized as the ultimate language user or the ultimate linguistic entity in India. An appreciation of the status of linguistics in India explains Bhartṛhari's frequent reference to Brahman in his writing--he was attempting to justify the tradition which led him to study language in the first place. The relation between that tradition and the formal models of language developed in earlier chapters is explored in Chapter V. It is my intent there to point to the formalism developed in connection with ākāṅkṣā and sphoṭa as forming a metaphor for Bhartṛhari's view of the relation between Brahman and the world. The reader will have to judge the success of the application. (A final Note: Śabda-Brahman 'speech-Brahman' is Bhartṛhari's term for Brahman as the ultimate linguistic entity. In Chapter V I shorten it to Brahman, as Bhartṛhari does frequently in his text.)

Throughout this volume my intent has been to render the concepts discussed clearly and precisely. To this end I have used some formal notation developed in the West. The sections of the dissertation which are primarily formal are described informally so that a reader who wishes to can read a prose account of what is done without undergoing the rigors of logical notation. While one may get through the research in this way, I do not believe that the subtleties of the formal rendering will be appreciated from the prose account. The formalism used in this description of my research seems to me to provide an understanding of the concepts formalized which is not possible to provide in ordinary prose. I do not mean by this that one is to view the formal"languages" used as distinct from ordinary

language; it seems to me that it is ordinary language sharpened into a formidable tool. (How this can be is one of the subjects treated in Chapter IV.) With the precision of the formalism comes an understanding of the nature and consequences of views first formulated less precisely--an understanding which I believe worth the trouble it takes to get the formalism right.

For information on the texts quoted from-- Pallai, Iyer, Raja, etc., the reader is referred to the Bibliography.

FOOTNOTES

1 For a fuller account of the history of the Grammarian school, the reader is referred to Misra (1966), pp. 11-28.

2 Chakravarti (1930), pp. 2-3. Chakravarti refers to the Taittirīya Saṃhitā VI.4.7 and the Mahābhāṣya, Vol. I p. 5 as texts.

3 I-tsing's account is reproduced in translation in Staal (1972), pp. 12-17.

4 Iyer (1969), p. 2. The primary evidence for the earlier date is the fact that Bhartṛhari is quoted in a Tibetan text of Diṅnāga, who is believed to have flourished circa 500 a.d..

5 See Iyer (1969), pp. 10-13. Iyer refers especially to a study by Kosambi which concludes "for all that, we still do not know who he was."

6 Pillai, pp. xv-xvii.

7 Iyer (1969), p. 6

8 See Chakravarti (1930), p. 34.

CHAPTER I

ĀKĀṄKṢĀ: THE SYNTACTIC BEHAVIOR OF WORDS[1]

1. Expectancy (Ākāṅkṣā)

The characterization of a sentence was a perennial subject of concern for the Indian philosophers of language. In Book II of the Vākyapadīya Bhartṛhari discusses eight definitions of the sentence which were in use when he wrote. In the next chapter his preferred view is considered. In this chapter we discuss a view he presented as an artificial way of getting to the truth--a view that sentences are sequences of words which have a certain property. According to this view, Bhartṛhari's definition of a sentence is the following:

(1) Bhartṛhari's loose definition of the sentence. A sentence is a sequence of words which expresses a complete meaning.

This definition subsumes another well-known definition of the sentence, that of the Mīmāmsaka school of philosophy:

(2) A sentence is one which has its parts possessing mutual expectancy when they are considered separately, but not possessing expectancy for anything else when in combination, which has the verb as its principal element, and has qualifying words and one unified meaning. (VP II 4, Pillai)

The last two parts of this definition were regarded as expendable, as we shall see, but the first part was considered to be the essence of the Mīmāmsaka theory. The notion of expectancy, or ākāṅkṣā, is described by Raja in Raja (1963) in this way:

(3) The Mīmāṃsaka-s first enunciated, and the other schools of thought later accepted with slight modifications, the three factors of phonetic contiguity or saṃnidhi, logical consistency or yogyatā, and syntactic expectancy or ākāṅkṣā among the parts of a sentence as constituting the bases of syntactic unity. Of these three, ākāṅkṣā is all-comprehensive and the most important. (p. 151)

(4) The knowledge of the syntactic unity of a sentence is mainly on the basis of the ākāṅkṣā, or the mutual expectancy, of the words. (p. 156)

(5) A word is said to have ākāṅkṣā for another if it cannot, without the other, produce knowledge of its inter-connection in an utterance. In every language certain words necessarily require certain other words to complete the sense. Thus a noun in the nominative case requires a verb to convey a complete meaning; a verb like 'bring' has expectancy for a word denoting some object. A string of words such as 'cow, horse, man, elephant' does not convey a complete sense, as there is no connection between them owing to lack of ākāṅkṣā. (p. 156)

A definition of a sentence based on ākāṅkṣā, then, would be this:

(6) The Mīmāṃsaka ākāṅkṣā definition of a sentence. A sentence is a sequence of words which lacks ākāṅkṣā.

The concept of ākāṅkṣā struck me early in my research as one which ought to be susceptible to formalization by means of a categorial grammar. (A categorial grammar is a grammar in which every expression is a member of a category. In particular, expressions of category A/B are the sorts of things which combine with expressions of category B to yield expressions of category A. In Mīmāṃsaka terms, an expression of category B/A has expectancy for an expression of category A.) I believe this impression to be correct. In Chapter II a categorial treatment of ākāṅkṣā for a fragment of Sanskrit is given and some examples of the principles employed are worked out. The result will be described in this section. In chapters to come the fragment will be used to make more precise the points under discussion.

The grammar in Chapter II discusses expressions of two languages. The first, Surface Level Sanskrit ("SLS"), is a fragment of Sanskrit as one might read it when the words are taken out of sandhi relationships. Correlated with the words of SLS are expressions of a language called Categorial Sanskrit ("CS") which have expectancies appropriate to those of the SLS words. If the SLS expressions correlated with a sequence of CS words may combine categorially to produce an expression of the category of sentences, then the grammar declares the SLS sequence a sentence.

Let us consider an example and its treatment in the style of the grammar for SLS:[2]

(7) Rāmas dhāvati 'Rāma' in nominative singular, 'run' in third person present singular active. 'Rāma runs'.

According to the grammar in Chapter II, Rāmas is correlated with a nominative noun phrase of CS and dhāvati is correlated with a verb phrase, an expression which takes a nominative noun phrase to make a sentence. Since an expression of the category of sentences results from their combination, (7) is declared a sentence.

The notion of ākāṅkṣā is modelled here by the creation of an unambiguous categorial grammar with which sequences of SLS words are correlated. If the CS expressions may combine to produce a CS expression of type S, then the SLS sequence is said to lack ākāṅkṣā--that is, to be a sentence.

There are many details to work out in making the model work, but it seems to me that it does work. The grammar in Chapter II makes precise the notion of ākāṅkṣā for the fragment of Sanskrit

which I treat. (There are many interesting features of Sanskrit which are not treated in the fragment. Later on some of them are mentioned.)

To my knowledge the system in Chapter II is the first grammar which provides a characterization of the sentences of a fragment of a natural language of comparable complexity by processing surface expressions. (The grammars with which I am familiar--Montague Grammars and Transformational Grammars--define the class of grammatical sentences by generating them from deeper structures in various ways.)

The SLS fragment is intentionally patterned on the fragment of English treated by Richard Montague in "The Proper Treatment of Quantification in Ordinary English" (Montague (1973)), hereafter "PTQ"). In PTQ Montague uses a categorial grammar[3] but employs expressions such as "a", "every", "such that" and others which are not expressions of the categorial language but are introduced syncategorematically.[4] It may be of interest to students of Montague's work to see that the grammar may be carried out in a strictly categorial way as well.

Students of Sanskrit may be interested to see some of the complexities involved in the treatment of what are superficially simple sentences of Sanskrit.

What follows in this section of this chapter is a description of the grammar in Chapter II together with an account of the use Bhartṛhari would make of it. The reader who wishes may skip the formalism of the chapter, although the full power of the categorial approach to grammar and the neatness with which it renders the ākāṅkṣā concept seem to me to be derived best from a perusal of the real thing.

Surface Level Sanskrit. The syntax of SLS is uncomplicated. There are six cases--nominative, accusative, instrumental, dative, genitive, and locative. There are the three genders masculine, feminine, and neuter. The lexicon contains the following common nouns and proper nouns in all six cases:

Masculine: go 'cow', nara 'man', ghaṭa 'pot', Rāma, Govinda, ta (pronoun), artha 'meaning', narasiṃha 'man-lion', and ya (pronoun).

Feminine: nārī 'woman', Lakṣmī, Çrī, ta (pronoun), ya (pronoun), and jāti 'universal'.

Neuter: gṛham 'house', ta (pronoun), ya (pronoun), pada 'word', vākya 'sentence', and dravya 'substance'.

All genders: bhavin 'entity'.

The following adjectives in all cases and genders are in the lexicon: sādhu 'good', mithyā 'false', cāru 'beautiful', satya 'real', and amithyā 'true'. (mithyā is declined here but not in natural Sanskrit.)

The operators ca 'and', vā 'or' and yadā...tadā 'if...then' are in the lexicon.

The operator na 'not' is in the lexicon.

The relativizer ta 'such that' is in the lexicon.

The determiner sarva 'every' is in the lexicon.

The verbs mṛ 'die', dhāv 'run', sthā 'stand' or 'exist', bhū 'exist', snih 'love', dṛç 'see', vac 'express', as 'be', yaj 'sacrifice', and yam 'give' are in the lexicon (most in both active and passive form) in third person present singular form.

Any sequence of words from the SLS lexicon is a SLS sequence. SLS sequences are correlated with expressions of CS, word by word.

Categorial Sanskrit. Some of the words of SLS are ambiguous between several forms. Gṛham, for example, is the nominative and accusative form of gṛham 'house'. As a nominative noun, intransitive verb phrases have expectancy for it to make a sentence; as an accusative noun, many transitive verbs have expectancy for it to make an intransitive verb phrase. Ta is a pronoun as well as the relativizer 'such that'. Cases of this sort are frequent and one difficulty encountered by the student of Sanskrit is that of choosing which of such alternative forms one uses in order to derive a sentence reading from a sequence of words.

CS is a language in which such ambiguities do not exist. The idea is that a SLS word with several possible expectancy readings is correlated with constructions of expressions of CS, each member of which represents one of the expectancies of the SLS word. "Gṛham", for example, is correlated with a nominative common noun expression and an accusative common noun expression. (There are others as well, as we shall see).

There is another sort of ambiguity found in ordinary Sanskrit which is treated in a similar way. It has to do with specifying the sense in which to take some occurrences of common nouns. Consider the following sentences:

(8) Naras dhāvati. 'man' in nominative, 'runs'

(9) Sādhus naras dhāvati. 'good' in masculine nominative, 'man', 'runs'

(10) Sarvas sādhus naras dhāvati. 'every' in masculine nominative, 'good', 'man', 'runs'.

Sentence (8) is ambiguous. The common noun naras may be taken in a definite sense or an existential sense, so that (8) may mean 'The man runs' or 'A man runs'. The sense of common nouns is often undetermined in Sanskrit. For this reason when expressions of SLS are correlated with CS expressions, common nouns are correlated with determiners corresponding to the two senses just described. These determiners will be separate from their common nouns, for reasons demonstrated by sentences like (9). 'The good man runs' and 'A good man runs' are possible readings of (9). In these cases we see that the adjective sādhus 'good' has modified the common noun naras before the determiner has operated on the common noun phrase. Hence determiners will be of a type with expectancy for common noun phrases. They are correlated with SLS common nouns but are autonomous and may delay combining with them so that adjectives may interact first.

Another sort of ambiguity occurring in Sanskrit is treated in a similar way. The Sanskrit sentence

(11) Çris sādhvī nārī. 'Çrī', 'good' in feminine nom., 'woman' in fem. nom.

means 'Çrī is a good woman' or 'Çrī is the good woman'. It is common for the verb as 'be' to be omitted. In the grammar in Chapter II as is introduced along with nominative nouns and common nouns in the set of expressions of CS correlated with SLS words.

Another sort of ambiguity arises from the use of pronouns. It is treated here is a different way. The Sanskrit sentence

(12) Rāmas Govindāya tasya ghaṭam yacchati 'Rāma gives Govinda his pot'

is ambiguous (whose pot is it?) In Sanskrit pronouns of any gender may have as antecedents nouns of the same gender. Ambiguity arises when there are several nouns of the appropriate gender to which a given pronoun may refer. In addition, there are ambiguities of scope of the sort treated in PTQ which were not, to my knowledge, discussed by the Grammarians. These facts--that pronouns may be ambiguous in reference, that scope may be ambiguous, and that pronouns refer to antecedents of their own gender--require some machinery in CS which is not a part of SLS.

Each pronoun of CS is an ordered triple whose last two members mark its gender and case. Its first member contains the pronoun form with a numerical subscript. The subscript serves to differentiate pronouns of the same gender for possible reference to different antecedents. The pronouns of the same subscript and gender may be taken to refer to the same antecedent.

Reference to an antecedent is brought about by the use of binding operators--operators which combine noun phrases and other phrases so that the noun phrase is related to pronouns of the proper subscript and gender in the other phrase. Each binding operator is subscripted; it binds only those pronouns which have its subscript. There are three categories of binding operators, representing these three scopes of the noun phrases which are the antecedents: sentence, verb phrase, and common noun phrase. The rules for combining binding operator, phrases containing pronouns, and noun phrases require that the noun phrase and the pronouns to be bound have the same gender.

The relation between noun phrase and pronouns is represented in the predicate calculus--the best tool for displaying the meaning of natural language expressions in a precise way I know of.

There is another sort of ambiguity not explicitly discussed by the Grammarians or the Mīmāṃsakas. The word vā 'or' may combine two noun phrases to make a noun phrase, two verb phrases to make a verb phrase, or two sentences to make a sentence. This fact requires that vā have three sorts of expectancy. In the grammar vā is associated with three expressions of CS, each with the relevant expectancy.

Basic CS Expressions. In CS we find the categories of common nouns, noun phrases, verbs (intransitive and transitive), determiners, adjectives, operators, relativizers, and binding operators. The notation of the system of categories is based in an obvious way upon that employed by Montague in PTQ. There are three basic types--CN, VP, and S, the types of common noun phrases, verb phrases, and sentences. Other category types are composed from these with the use of slashes (as in PTQ, "A/B" is used to denote the category of expressions which take things of type B to produce an expression of type A.)

Common Noun expressions are ordered triples whose second and third members are gender and case markings. The first member contains a predicate followed by "(z)". For example, the Common Noun expression correlated with rājaṣ 'king' in nominative is ⟨raja(z),M,Nom⟩. The "z" holds the place for an argument which may be inserted when the expression is combined with another.

Noun Phrases are members of category S/VP. They are ordered triples whose second and third members are gender and case markings.

Their first member is a constant or variable enclosed in parentheses preceded by the letter "P". The noun phrase expression correlated with Rāmeṇa 'Rāma' in instrumental is ⟨P(Rāma),M,Inst⟩, for example. The "P" holds the place for a predicate expression which may be inserted when this expression is combined with another.

Verbs are members of three categories, intransitive verbs (category VP for verb phrase), 1-place transitive verbs (category VP/NP), and 2-place transitive verbs (category (VP/NP)/NP). Verbs are n-tuples whose first member is a predicate expression with markers which specify the case each of its arguments must be in. The other members are the cases of the arguments in reverse order in which they are to be filled in. For example, correlated with the verb yacchati 'give' in active is the CS expression ⟨yam Nom Acc Dat, Nom,Acc,Dat⟩. The first member is the predicate yam 'give' with its three argument places marked with names of the cases its arguments must be in. The last three members show that the first argument to be taken will be in dative case, the second accusative case, and the third nominative case.

Passive forms of verbs are treated as synonymous with active forms.[5] The passive of yam, yamyate, is correlated with two CS expressions, ⟨yam Inst Acc Nom,Nom,Acc,Inst⟩ ('___was given a___ by___') and ⟨yam Inst Nom Dat,Nom,Dat,Inst⟩ ('___was given to___ by___'). The result of combining any of these three versions of yam with arguments will be an expression of the form yam a b c, where a is the giver, b the gift, and c the recipient.

There are some interesting features of the treatment of the verb yaj 'sacrifice'. The sentence

(13) Rāmas gavā narasiṃham yajati

which means

(14) Rāma sacrifices a cow to a man-lion.

may be true even though no man-lions exist. This is not the case with a sentence such as

(15) Rāmas gām narasiṃhāya yacchati

'Rama gives a cow to a man-lion'

which is never true when there are no man-lions. Put crudely, it is Rāma's intention rather than the existence of man-lions which counts in determining the truth value of (13). Hence yaj is a predicate which has an intensional third place. This fact is represented in the fragment by correlating with yajati, the active form of yaj, the CS expression ⟨yaj Nom Inst ^Acc,Nom,Inst,Acc⟩. The symbol "^" is an intension operator, after the style of Montague in PTQ. It should be noted that it need not mean what it means in PTQ, however. According to Montague's interpretation it is the extension of the expression which follows it, in all worlds and times. In a

faithful rendering of Bhartṛhari's theory, it would probably denote some concept of the object of the sacrifice which the sacrificer has in mind, but I forego pronouncing on this point until a later chapter.

The student of PTQ may notice that the intensional operator is introduced as part of the verb expression rather than, as in PTQ, as a built-in feature of the categorial combination process. This point is elaborated in discussion of the rules of combination.

The category of determiners has three members, corresponding to the definite description determiner (the man), the existential quantifier (a man), and the universal quantifier (every man). Determiners are ordered triples whose second and third members are gender and case markers and whose first member is a quantified expression containing an occurrence of "CN" to hold the place for a common noun predicate when the determiner and common noun are combined. The determiner correlated with sarva 'every' in masculine nominative, for example, is $\langle (x_{2i})(CN(x_{2i}) \rightarrow P(x_{2i})), M, Nom \rangle$, where i is the number of sarva's place in the SLS sequence. This expression will have the "CN" filled in by a common noun predicate and the "P" filled in by a verb.

Binding operators are of the form $VPBO_i$, $CNBO_i$, and SBO_i. The first quantifies noun phrases into verb phrases, the second into common noun phrases, and the third into sentences. The categories of these operators are VP/(NP,VP), CN/(NP,CN), and S/(NP,S).

Adjectives are ordered triples whose first member is a stem and whose second and third members are gender and case markers. The

SLS adjective sādhus 'good' in masculine nominative, for example, is correlated with the CS expression ⟨sādhu,M,Nom⟩. Adjectives are of category CN/CN.

Relativizers are of the form $\langle Rel_i,G,C\rangle$, where the first member has as subscript the integer which is the subscript of the ya pronouns it will bind, and the second and third members are gender and case markers. These expressions are of category CN/(S,CN).

Category S/(S,S) contains the logical operators ∨, &, and →.

Category S/S contains the logical operator ~.

The category VP/(VP,VP) contains the verb phrase disjunction and conjunction—∨ and &.

The category NP/(NP,NP) contains the noun phrase disjunction ∨.

There are other members of the categories of CS in addition to these basic ones just described. In order to explain their derivation we consider the Correlation Rules and the Rules of Combination.

Correlation Rules. SLS words are correlated with sequences of CS expressions which reflect their expectancies. The Correlation Rules are as follows:

Each common noun is correlated with a set of sequences of CS expressions which is the union of the following two sets (taken for nārī in feminine nominative, for example):

1. $\{\langle nārī,F,Nom\rangle\}\ \underline{X}\ \begin{Bmatrix}\emptyset\\ as\end{Bmatrix}\ \underline{X}\ (Det \cup \{\emptyset\})$

2. $\{\langle nārī,F,Nom\rangle\}\ \underline{X}\ \begin{Bmatrix}\emptyset\\ as\end{Bmatrix}\ \underline{X}\ Det\ \underline{X}\ \{\langle ta_{2i},F,Nom\rangle\}\ \underline{X}\ \begin{Bmatrix}NPBO_{2i}\\ VPBO_{2i}\\ SBO_{2i}\end{Bmatrix}$

The first member of the first set is the expectancy of nārī proper. The second represents the possible addition of the verb as 'be'. The third represents the possible addition of a determiner. The operator "X" is used to denote the Cartesian product of the sets it flanks, with "Ø" (the empty set) deleted. Thus the first set includes $\{\langle P(n\bar{a}r\bar{\imath}),F,Nom\rangle\}$ if "Ø" is the element in the product selected from the second and third set. The second set is like the first except that a pronoun and binding operator are added with the determiner in order to allow for varying scope on the quantifiers. As before, "i" is the number of nārī's place in its SLS sequence. If nārī had not been in nominative case, the terms containing as would be deleted. (We recall that as 'be' takes nominative case arguments.)

I have stated the correlation rules in this way to make perspicuous the source of each member of a correlated sequence. It would have been less complicated but a more lengthy process to have specified the set of correlated expressions one by one. Instead I have opted for the empty set notation and modified Cartesian Product operator, in the tradition of the Grammarians themselves, to whom brevity in the statement of rules was a goal pursued with cleverness.

The set of sequences of CS expressions correlated with a proper noun is less complicated. For Rāma in nominative, it is $\{\langle P(R\bar{a}ma),M,Nom\rangle\}$ X $\left\{\begin{matrix}\emptyset\\ as\end{matrix}\right\}$.

The set of expressions correlated with ta, in neuter genitive, for example, is the union of these sets, where ta is the second

member of a SLS sequence of length 3:

1. $\{\langle P(ta_2),N,Gen\rangle, \langle P(ta_6),N,Gen\rangle\}$

2. $\{\langle P(ta_4),N,Gen\rangle\} \underline{X} \begin{Bmatrix} \emptyset \\ VPBO_4 \\ CNBO_4 \\ SBO_4 \end{Bmatrix}$

3. $\{\langle rel_4,N,Gen\rangle\}$

(If ta had been in nominative case, sets 1 and 2 would have had the term $\{\emptyset, as\}$ added to the product.)

The first set represents ta as a pronoun which may be bound by a binding operator introduced somewhere else, or which may remain unbound, referring to some antecedent outside the sentence. The second represents the pronoun with its own binding operators. The third represents ta as a relativizer.

The pronoun ya in genitive neuter, as second member of a three-member sequence, would have correlated with it this set:

$\{\langle P(ya_2),N,Gen\rangle, \langle P(ya_4),N,Gen\rangle, \langle P(ya_6),N,Gen\rangle\}$.

Each pronoun serves as a marked noun phrase until a relativizer of its subscript combines a sentence-category expression containing it and a common noun to make a common noun.

The other combination procedures are straightforward, save that for the word yadā...tadā 'if...then' a single expression is correlated. This expression, "→", is of a category S/(S,S). There are other ways this expression could have been treated. Montague might have inserted it syncategorematically, making if p then q directly from p and q in PTQ. There are no syncategorematic words in this fragment, but we could have taken one of the two "words" (yadā, for example) to be of category (S/S)/S, and taken the other to be of category

S/((S/S),S). But there is no evidence I am aware of which suggests that this is the way if...then sentences are built up in practice, and in the absence of such evidence the simpler tack adopted here was taken.

Given a sequence s of SLS words, the sets of expressions correlated with each word of s are combined in all possible ways to produce the set of disambiguated sequences for s. Members of such sequences may combine to produce derived expressions of the categories of CS by means of the Rules of Combination.

Rules of Combination. There are several departures here from the methods employed by Montague in PTQ. One is that most of the Rules of Combination are partially, rather than totally, defined, depending on factors such as agreement in gender and case in the selection of appropriate arguments. This is not a necessary feature of the system. One could make the rules be totally defined and preserve the relevant features of the system by adding a place to each CS expression which is "W" for basic expressions but which changes to "I" whenever an inappropriate sort of expression is operated on to produce a combined expression. Then the definition of a sentence would have the provision added that the second member of the thing of type S produced by the interaction of the members be "W" (well-formed) rather than "I" (ill-formed). I have not adopted some such maneuver here because there are semantic reasons for denying the total-function nature of categorial expressions (at least, as they are employed in PTQ, where Montague allows vacuous quantifying in and derives some spurious readings.) Also, the

grammar of Chapter II is an attempt to model a listener's or reader's processing of Sanskrit. The listener or reader may try all sorts of interpretations of words in order to arrive at a sentence reading, but a competent listener does not combine things which do not agree in case or gender.

Some features of the Rules of Combination are the following:

Common nouns and determiner combine when gender and case agree to produce a noun phrase.

Noun phrases and verb phrases combine when the noun phrase is in nominative case to produce a sentence.

1-place transitive verbs combine with noun phrases in the correct case to produce verb phrases.

2-place transitive verbs take noun phrases in the correct case to produce 1-place transitive verbs.

Common nouns and adjectives combine if case and gender agree.

A noun phrase is quantified into a common noun phrase, verb phrase, or sentence by a binding operator when the phrases quantified into contain at least one pronoun with the same gender as the noun phrase and the same subscript as the binding operator, and when all such pronouns in the phrase quantified into have the same gender as the noun phrase.

Relative clauses in Sanskrit are somewhat different in construction from their English counterparts. In Sanskrit the common noun phrase to be relativized is marked with the relativizer ta, which agrees in gender and case. Correlated with ta is at least one occurrence of ya in the relative clause, agreeing in gender, but

marked with a case which depends on its function in the relative clause. The combination rule for relativizers, sentences, and common nouns, then, states that a common noun and sentence are combined by a relativizer to make a relative clause when there is at least one occurrence of a ya pronoun in the sentence with the same subscript as the subscript of the relativizer, where every such pronoun in the sentence is of the same gender as that of the common noun phrase, and where the gender of such pronouns matches the gender of the relativizer and the common noun. For example, in the SLS sentence

(16) Rāmas gām tam Çriyā yamyate yas dhāvati 'Rama', 'cow', 'such that', 'by Cri', 'is given', 'it','runs'

'Rāma is given a cow which runs by Crī'.

gām 'cow' is in accusative case in the main clause and is marked by the relativizer tam in accusative case. The pronoun yas bound to it is in the nominative case in its clause. The rule combines tam, gām, and the phrase yas dhāvati to produce a relativized common noun--cow which runs.

The way in which first members of derived expressions are formed is worth comment. In PTQ when an expression a and an expression b are combined, the usual translation of the result is what is derived from applying the translation of a to the intension of the translation of b. As a result Montague was forced to use Meaning Postulates to yield equivalent extensional readings for those verbs which have an extensional sense--the majority of verbs. For some combinations of elements--quantifying in and relative clause formation for example--lambda operators are used in order to yield expressions which may

be transformed through rules of lambda conversion into predicate calculus expressions which are the natural readings of the sentences. It has not seemed to me that the intensional expressions and Meaning Postulates needed to derive extensional readings for them are the best way to represent the translations of natural language fragments. For one thing, there is no feature of my language processing which I am able to discover corresponding to the change from intensional to extensional expressions with the aid of Meaning Postulates. For another, the lambda notation in which the PTQ translation of English expressions is given is, as far as I can discover, much more complicated than my understanding of those expressions. For these reasons it has seemed worth exploring ways to present the translation of CS expressions in predicate calculus notation directly, with the intensional operators built into the translation of the intensional expressions (yaj, for example) and the effect of using lambda notation and rules of conversion built into the Rules of Combination. I believe the grammar in Chapter II to be the successful result of such exploration. There a translation into predicate calculus is the first member of every expression. Expressions not of category S have markers to hold the place for expressions with which they will combine. Such markers are "z" for arguments of common nouns, "P" for properties, "CN" for common noun predicates, and the various case markers for verb argument places.

Let us consider an example. The compound noun phrase Rāmas Çrīs vā of SLS ('Rama or Cri') has this CS expression correlated with it:

(17) $\langle P(\bar{R}ama) \vee P(\acute{C}r\bar{\imath}), N, Nom \rangle$

The verb dhāvati 'runs' has this CS expression correlated with it

(18) ⟨dhāv Nom,Nom⟩.

RC2, the second Rule of Correlation, describes the expression derived from the combination of these two as $F_4(P(\bar{R}ama) \vee P(\acute{C}r\bar{\imath}), dh\bar{a}v\ Nom, Nom)$, where F_4 is a function which does what the rules of lambda conversion do: it puts the predicate dhāv in the place of P in the expression $P(\bar{R}ama) \vee P(\acute{C}r\bar{\imath})$, while putting Rāma and Çrī in the place of Nom in the expression dhāv Nom. The result of applying F_4 to the three expressions above is

(19) $dh\bar{a}v(\bar{R}ama) \vee dh\bar{a}v(\acute{C}r\bar{\imath})$

This expression is naturally read as "Rāma or Çrī runs".

This way of handling translations seems to me to have advantages beyond that of directness. The characterization of ākāṅkṣā in (5) specified that a verb has expectancy for a word denoting some object, and a nominative noun has expectancy for a verb. The categorial notation accounts for the first statement but not the second. I believe it is an explication of the second that the translation of a nominative noun phrase has a place held by "P" in which a predicate is to be inserted. The translation shows that the expression "expects" a verb in order to be complete.[6]

Examples of SLS sentences and the combination of their disambiguation sequences to expressions of type S are found in Appendix A, where it is demonstrated that the following sentences:

(20) Çrīs gṛham paçyati.

(21) Sarvas naras nāri snihyati.

(22) Rāmas tam grām Çriyā yamyate yas dhāvati.

have the translations, respectively,

(20') $(Ex_5)(gṛham(x_5)\&drç\ \acute{C}r\bar{\imath}\ x_5)$

'There is a thing which is a house and Çrī sees it.'

(21') $(x_3)(nara(x_3)\rightarrow(Ex_7)(n\bar{a}r\bar{\imath}(x_7)\&snih\ x_3\ x_7))$

'For every man there is a woman whom he loves.'

and

$(Ex_7)(n\bar{a}r\bar{\imath}(x_7)\&(x_3)(nara(x_3)\rightarrow snih\ x_3\ x_7))$

'There is a woman who is such that every man loves her.'

(22') $(Ex_7)((go(x_7)\&dh\bar{a}v(x_7))\&yam\ \acute{C}r\bar{\imath}\ R\bar{a}ma\ x_7)$

'There is a thing which is a cow and runs and Çrī gives it to Rāma.'

Given these rules the definition of a SLS sentence follows simply.

The Definition of a Sentence of SLS. Informally, an SLS sentence is a sequence of SLS words which is susceptible to interpretation so that its expectancy is satisfied. There is one other provision: no occurrence of ya must remain when expectancy is satisfied. (This was not part of the Mīmāṃsaka definition, I believe, because their attention was not drawn to quantificational phenomena.)

More formally, the definition is this:

(23) A SLS sequence s is a SLS sentence if and only if there is a member of the set of disambiguated sequences for s such that some sequence of application of the Rules of Combination to that member transforms it into a single CS expression of type S which contains no expression of the form $\langle ya_i, G\rangle$.

It may seem that the grammar is deficient in one important respect, in that it generates many more sequences to test in the

set of disambiguated sequences for a SLS sequence than will end up producing an expression of type S. The set of disambiguated sequences correlated with (20), for example, contains 36 members, only 6 of which may produce expressions of type S, and the translation of each of those 6 members is identical. For (21), there are 1,296 members, most of which do not produce an expression of type S. Hasn't the grammar overgenerated sequences?

In a sense, the answer to this question is "yes", for no reasonably experienced reader of Sanskrit supplies as 'be' with a nominative noun phrase when there is already a verb to take that noun phrase as subject. Similarly, one doesn't supply determiners for a common noun if it has already got one. Moreover, one doesn't worry about different binding operators with different scopes if there is only one quantifier in the expression being translated. The grammar in Chapter II represents the procedures used by a reader of Sanskrit who has not yet learned some very obvious strategies for cutting down the possibilities of interpretation. This fact does not constitute a criticism of the grammar, however, for it does correctly represent the brute force approach of an untutored reader. Furthermore, it is a simple enough matter to formulate interpretational strategies and work them into the Correlation Rules. (The proviso that the term containing as is deleted from the products in CR1 and CR2 is such a strategy). There are many ways this might be done and, as this topic doesn't bear directly on the concept of ākāṅkṣā, I have left the Correlation Rules for the most part in their unsophisticated and unconstrained form.

2. Comments on the Fragment

There are several aspects of the Mīmāṃsaka theory which have not been correctly modelled in the fragment found in Appendix A. Some of them are the following.

Imperative form. In the Mīmāṃsaka system (and in Vedic-oriented philosophy of language in general) the paradigm form of a sentence is that of an imperative, whereas in the fragment given here there are only indicative sentences. There are three reasons for this alteration in the theory. First, the syntactic characterization of sentences as sequences of words with expectancies which are satisfied seems to apply as well to declarative sentences as to commands. In this respect it doesn't seem to matter what sort of sentence one treats (many of the examples given in the disputes between the Mīmāṃsakas and Bhartṛhari are indicative rather than imperative.) Second, Bhartṛhari deals as often with declarative sentences as imperatives, and the Mīmāṃsaka concentration on imperatives does not seem to be a fixation which Bhartṛhari endorsed. Third, much of my interest in modelling the Indian theories of language in formal ways lies in examining the interaction between the insights of Western analytical philosophical researchers and those of the Indian semanticists. Western philosophers have concentrated most heavily on the form and truth conditions of the declarative sentence, and their results are most fully worked out there. Hence I treat declarative sentences with the Western formalism in order to use the best Western technology. (Montague suggests, in a footnote to PTQ, that "In connection with imperatives and interrogatives

truth and entailment conditions are of course inappropriate, and would be replaced by fulfilment conditions and a characterization of the semantic content of a correct answer." (p. 248) Montague did not carry out such a characterization before his death, however, and I know of no well-developed system which has done so.)

Ākāṅkṣā as a Syntactic Notion. In developing the notion of ākāṅkṣā as a syntactic one, I have deliberately played up the syntactic characterizations of ākāṅkṣā by the Mīmāṃsakas and played down those characterizations which involve semantic notions. Here are some passages of the semantic sort:

(24) ...it is possible to assert that a substantive, a declensional suffix, a radical stem, and a conjugational suffix,--all these are mutually expectant, having semantic interdependence on each other for the communication of a particular relational thought-unit... (Bhattacharya (1962), p. 138)

(25) There (in the Kātyāyanaśrautasūtra) a sentence is described as that which is nirākāṅkṣā, that is to say, something which has no requirement or expectation of words outside itself to complete its meaning. (Raja (1963), p. 154)

But the talk about semantic interdependence and meaning in these passages seems to me to be derivative rather than primary. That is, in (24) the syntactic features discussed are required in order for the sequence of words to be capable of communicating a thought-unit. Hence the necessary feature is a syntactic one, not a semantic one. My reading of (25) and passages like it is similar. The expectation itself is given in examples as syntactic, although there is a temptation for the commentators to describe the expectancy as semantic. Some reasons for that tendency are discussed in a later chapter.

It is interesting to note that the syntactic characterization is sufficient to determine the class of sentences of the fragment, and that in fact no semantic features are used in the definition of an SLS sentence. (The first member of each CS expression need not be its "translation". The system would work just as well with any way of forming first members which kept track of ya and ta for later binding.) In this respect the ākāṅkṣā definition resembles the characterization of sentences given by contemporary Transformational Grammarians, whose goal is sometimes described as the specification of the class of well-formed sentences using only syntactic notions. In fact, the fragment in Chapter II is strongly equivalent to a Transformational Grammar given in Chapter II. (This grammar was suggested by the results in Cooper and Parsons ( ).) In making the Mīmāṃsaka definition a syntactic one, then, I have at least put them in respected company.

Other Phenomena. There are many other features of Sanskrit syntax discussed by the Indian philosophers of language which would have been a delight to treat here but which have been omitted because they are not relevant to later sections of this work. Some of these features are:

Compounds. There are three sorts of compounds in Sanskrit, each interpreted differently, and each with its own grammatical peculiarites. A good deal of disputation in India centered on the nature of the process by which the meanings of words interact in producing the meaning of a compound. An adequate model of compound formation would throw light on those disputes (or vice versa).

Tense and Number. Western theories of past and future tenses and plural number are not as neatly worked out as are the theories for sentences with singular number and present tense. Hence I have not treated these phenomena here, although there is a wealth of interesting data to treat.

Locative Absolute (and Genitive Absolute). There are Sanskrit formations in which the subject and participle, as well as modifiers, are in locative case. Such absolute constructions are used to describe actions which have already taken place. I do not know what the best way to model such constructions would be (at the least, a theory of tenses is required for such a model) but it would appear to involve substantial complications in the grammar. There is one interesting feature of the genitive absolute, in that a semantic restriction on grammaticality is given for it by Whitney in Whitney (1889):

(26) The genitive always indicates a living actor... (p. 101)

If this restriction is correctly described, the grammar could not be strictly based on syntactic notions.

Further Noun Phrase Ambiguity. Noun phrases have more uses than those given in the fragment. Rāmeṇa 'Rāma' in instrumental, for instance, means 'with Rāma' as well as 'Rāma' in a case which may be taken by a verb. Such uses would be represented by additional members of the sets of expressions correlated with common nouns and noun phrases, but to add them here would complicate the model without clarifying the issues to be discussed later on.

Adverbs, Prepositions, and Particles. An adequate model for linguistic theories of the Indians would include adverbs, prepositions, and particles of various sorts, so that different theories about the number of parts of speech might be illuminated. (In particular, the dispute of some Grammarians over the classification of particles called karmapravacanīyas as separate sorts of words in addition to noun, verbs prepositions, and particles would be illuminated. Bhartṛhari has a theory about the special status of karmapravacanīyas, but to make it out more precisely would require a grammar with various sorts of particles in it, as well as tensed expressions.

Yogyatā (fitness). We noted in the previous section that the Mīmāṃsaka definition of a sentence quoted in (3) involved three factors, two of which were ākāṅkṣā and yogyatā (fitness). Yogyatā has not been a feature of the grammar in Chapter II. Why is this?

It is not a necessary feature of the model that it leave out yogyatā, but Bhartṛhari and the Mīmāṃsakas of his time seem to have given it up as a requirement which ought to be imposed on sentences, and I believe their reasons are good ones. Let us discuss them in detail.

The notion of yogyatā is characterized in two passage which we consider:

(27) Do the words 'Fire is cold' constitute a proposition? It must be admitted that there is the required syntactical expectancy...but is it in reality a valid proposition? The answer from a purely commonsense point of view would be in the negative. But why...Because there is no syntactical possibility between the constituent concepts, the inherence of 'coldness' in 'fire' being physically impossible.
(Bhattacharya (1962), p. 140)

(28) In the sentence 'He wets it with water' (payasā siñcati), there is yogyatā or consistency of meaning, since wetting is generally done with a liquid like water, and there is nothing incompatible between the idea of wetting and that of water. But a sentence like 'He wets it with fire' (agninā siñcati) has no yogyata or compatibility, since the idea of wetting is something incongruous with that of fire...Strictly speaking it is the inconceivability of the mutual association of the word-meanings that renders the whole sentence nonsensical; it is not the lack of correlation with the actual facts, but the impossibility of connecting the word-meanings that stands in the way of verbal comprehension. (Raja (1963), pp. 164-5).

The idea expressed in (27) and (28) seems to be that "Fire is cold " and "He wets it with fire " are sequences of words which are unfit to be ruled sentences. While they satisfy the requirement of ākāṅkṣā, the condition of having proper fitness is not satisfied by them. There has been some Western concern over such sentences. Noam Chomsky in his first book, The Syntactic Structure of English, asserts that sentences such as

(29) Colorless green ideas sleep furiously.

are syntactically well-formed. In his Aspects of the Theory of Syntax Chomsky seems to have given up this position, adopting a theory in which words have selectional restrictions on those words with which they combine. An inanimate subject, for example, would not be combined with a verb which requires animate subjects to produce a grammatical sentence in this theory. In Aspects, then, Chomsky seems to uphold the requirement of yogyatā on sentences. Barbara Partee informs me that Chomsky may have reverted to his earlier position at present, giving up this requirement. Most of the Indian philosophers of language at the time Bhartṛhari wrote had done the same. There are various reasons for this. One is

that agreement on cases is hard to find. (Many people who are sympathetic to a yogyatā requirement on sentences would nonetheless call "Fire is cold " a false sentence, since "Fire is hot " is true and the idea seems to be that hot and cold have fitness for the same sorts of things.) Another is that many people would declare both sequences given as non-sentences in the passages we have quoted to be false, which is something one can not do if the sequences are not sentences.

Some Western semanticists hold a theory which seems similar to the yogyata theory. In Thomason (1972), Richmond Thomason gives this as an example of a sentence which lacks truth-value:

(30) The theory of relativity is shiny.

The idea is that the theory of relativity is not the sort of thing one may truly (or falsely) predicate shininess of, and a sentence in which such a predication is made is one which lacks truth value. What (30) lacks is adequate presupposition, rather than fitness.

If we read "lacks truth value" or "has presuppositional failure" for "lacks fitness", then we find a modern counterpart of the Mīmāṃsaka theory of yogyatā in the theory of presuppositional failure. A formal model for such a theory has been hinted at by Thomason in Thomason (1972) and worked out in detail by James Waldo in Waldo (1977). The difficulty with taking the yogyatā theory in this way, however, is that to do so is to read sequences of words which lack yogyatā as sentences which lack truth value--that is, as sentences.

(Thomason gives a powerful argument for ruling (30) a sentence. To deny sentencehood to (30) involves denying it to

(31) What Rāma is thinking of is shiny.

when what Rāma has in mind is the Theory of Relativity, and allowing it to (31) when what Rāma has in mind is a gold ring. It seems peculiar that the grammaticality of a sequence of words might vary from moment to moment, whereas there is nothing untoward about its truth value so varying.)

Saṃnidhi or Āsatti (Contiguity). The third requirement contained in the classical Mimamsaka definition of a sentence is saṃnidhi (sometimes, āsatti), that of 'contiguity'. There are two distinct readings of this requirement in the literature. I do not believe either should be part of a characterization of sentences of Sanskrit.

The first has to do with temporal contiguity.

> (32) Saṃnidhi or āsatti is generally explained as the condition that the words in a sentence should be contiguous in time. Words uttered at long intervals cannot produce the knowledge of any interrelation among them, even if there be ākāṅkṣā or yogyatā. (Raja (1963), p. 166)

But there are instances in which words separated by long intervals are taken to be parts of a sentence. Messages shouted to someone in an echo chamber, for example, might be given a word at a time, with intervals to allow the previous echoes to die away. One can imagine more bizarre cases--messages broadcast word by word from behind erratically interfering asteroids, for example--in which the intervals between words might be quite large, yet the words would be taken to be constituents of a sentence. In fact, it seems that this requirement is one on a person's grasping of a sequence of words as

a sentence, rather than on the sequence's being a sentence, and should properly be a part of a theory of language processing rather than a theory of what language is.

The other interpretation of sannidhi has a good deal of comment, both ancient and modern, surrounding it. First, the notion itself:

> (33) If the words are separated by the intervention of irrelevant words, then also the connection of the meaning cannot be understood. (Raja (1963), p. 166)

> (34) The next item of contiguity (āsatti) now comes up for discussion. The intended relational cognition cannot arise from a group of words if the terms expressing concepts expecting one another be separated by intervening words not having any immediate syntactical expectancy with the former....For example, when we say--"The mountain has eaten is fiery Devadatta", meaning thereby that "The mountain is fiery" and that "Devadatta has eaten", there is the absence of contiguity inasmuch as there is temporal intervention between the interdependent concepts due to the utterance of other words in between so that the emergence of the intended judgmental cognition is obfuscated.
> (Bhattacharya (1962), pp. 153-4)

The reader may have noticed that the grammar given in Chapter II allows for complete freedom of word order. That is, if some sequence of SLS words is a sentence, then any permutation of the members of that sequence is also a sentence. This feature of the model denies the claim made in (33) and (34) that words separated by words which do not combine with them fail to produce a sentence. In fact, given the relevant additions to the lexicon of SLS and the addition of "and" to the sequence in question so that the example is one of two sentences conjoined, the sequence discussed in (34) would be declared a sentence by the grammar of Chapter II. This feature of the model also denies the claim made by many contemporary Western linguists that certain sorts of phrases are inviolate--that tensed

sentences, for example, may not contain words from other phrases in a grammatical sentence. This constraint is typically taken to be one which holds of all languages, including Sanskrit.

But there is evidence that the contemporary Western grammarians and the samnidhi theoreticians are wrong. This conclusion has been argued for by many Indian philosophers of language, and the controversy has been summed up most neatly by J.F. Staal in Staal (1967):

(35) These discussions show that, even if there is some uncertainty with regard to word order in the utterances of the Veda, the Mīmāṃsa philosophers agree unanimously that word order in ordinary language is free. (p. 47)

(36) Summarizing, we may conclude that Western Sanskritists started to describe a traditional, habitual or merely preponderant fixed word order in Sanskrit. When these investigations became more precise, more exceptions were met with and various habitual arrangements of words for various kinds of sentence were postulated. It was also increasingly realized that word order has no grammatical significance or value. But most of the investigations by Western Sanskritists were based upon a specifically selected corpus of Sanskrit texts. (p. 59)

(37) Summarizing, we may conclude that the Indian theorists agreed almost unanimously that word order in Sanskrit is free. (p. 49)

What Staal is summarizing here is the later views of the Indian philosophers of language, Mīmāṃsakas and Grammarians alike. They seem to me to be correct.

Of course, assertions that word order is free in Sanskrit do not prove the point, and it is a point which would be difficult to prove conclusively. Instead, Staal cites counterexamples to plausible candidates for word order principles of Sanskrit in Staal (1967). Since the tensed sentence constraint is used by contemporary linguists, it may be of interest to see what sort of sentence

counterexamples it. The sequence of SLS words

(38) dhāvati, naras, Çrīs, Sādhus, ca, dhāvati

'runs' 'man' 'Cri' 'good' 'and' 'runs'

'The good man runs and Çrī runs

has the adjective sādhus 'good' in masculine nominative after the feminine subject Çrīs of the second clause. Such usage might come as the result of the author's intention to keep the goodness of the running man in the reader's mind while Çrī is talked about. According to Indira Shetterley (my referee for such judgments) this is a poetic device rather than the sort of construction one would find in good Sanskrit prose. However, given the continuum between poetry and prose in Sanskrit usage, perhaps Staal's decision to regard word order as completely free is correct. If the tensed sentence constraint is indeed a constraint on acceptable prose sentences of Sanskrit, one could impose it on the grammar readily enough, in a fashion to be discussed soon. I have assumed for the purposes of this chapter that it is not.

According to Indira Shetterley, the sentence

(39) Rāmas karoti gām ghatam Govindas ca paçyati

'Rāma' 'makes' 'cow' 'pot' 'Govinda' 'and' 'sees'

has as preferable reading 'Rāma makes a pot and Govinda sees a cow'--a reading which would not be allowed by the tensed sentence constraint. She says, "If I saw this sentence in a poetical passage I would derive a good deal of pleasure from untwisting it. If I saw it in prose, I would become angry". The fragment, then, is a poetical one. To make it proselike, one would only have to stipulate that trees for the Transformational Grammar representation of such sentences not have lines

from members of different constituent sentences which cross.

There are some principles of ordering which I have not found to be violated in my reading:

Absolute constructions are not invaded by words from the main sentence, even in poetry.

Words followed by the quotation operator iti are taken as a block, and no intrusion is allowed.

ca 'and' and vā 'or' occur anywhere to the right of the first words of the second member of the two phrases they operate on. That is, ca and vā seem to move freely only to the right.

The first two principles involve what may be the construction of wordlike fragments from words, and it may be possible to view absolutes and quoted fragments as superwords of some sort. As for the third, although I have not found it violated, I do not know whether it can be violated and so have not imposed it on the fragment.

There are many respects in which the fragment fails to be a natural rendering of some version of Sanskrit or other. I have added words (the declined mithyā, for example) to the language and altered the meanings of other slightly ('give' for yam instead of 'check' or halt', the preferred readings). Nonetheless, the fragment seems to me to be of interest in the same way that Montague's fragment in PTQ is of interest: it is a start toward more sensitive renderings of natural language, and it includes treatment of some puzzles which must be solved if such renderings are to be formulated. There are strained readings and infelicities, but the theory rather than the weak applications is what interests me here.

Rules and exceptions. Bhartṛhari's characterization of the sentence differs from the Mīmāṃsaka characterization in the respect that his treatment of rules and exceptions to them requires a different analysis of the ākāṅkṣā of exceptions. Consider this passage:

> (43) It is considered by some that a rule and an exception form one sentence (statement) even if they have several verbs. Only, they appear to be different sentences.
>
> Thus a restriction or prohibition forms part of the general injunction;...
>
> (Here the upholder of the doctrine that such sentences are really distinct intervenes):--
>
> When the sentences have no expectancy (for anything outside) and are at rest so to speak, they are independent of one another. Therefore, since there is the absence of a relation of one being for the sake of the other, how can they together form one sentence?
>
> (The upholder of the view that the two form one sentence replies):--
>
> A special rule causes the remaining part of a sentence (i.e., the prohibitory sentence here) to be inferred because it (the special rule) needs such inference. Therefore, there is in the object to be prohibited as much expectancy as there is in the object of the special rule. (VP II 348-351, Pillai)

Although the proponents of these two points of view are not identified in the Vākyapadīya, Pillai attributes the view that a rule and its exception form a single sentence to Bhartṛhari and he attributes the other view, that the two are distinct sentences, to the Mīmāṃsakas. This difference of opinion is not treated in the fragment of Chapter II because I do not know what the best way to formalize Bhartṛhari's "special rule" would be, although I believe there are good reasons for holding his position on this matter. Consider these two sentences:

(44) Everyone may come to the wedding.

(45) Warriors are the only exception to (44).

If we take (44) and (45) to be distinct sentences, it seems that they must also be taken to yield a contradiction, for the meaning of (44) is that all individuals are invited to the wedding, and (45) means that warriors are not. Surely this is not the correct way to treat them. A person who gives a rule followed by an exception often knows that the exception is to follow the rule; there is no sense of self-contradiction in this knowledge.

The problem of exceptions is not confined to injunctions, as these sentences demonstrate:

(46) Everyone entered by the front door.

(47) The sole exceptions were tradesmen.

Barbara Hall Partee has pointed out that the problem arises within single sentences as well. For example, in

(48) Everyone shot arrows except Govinda.

a compositional semantics would yield a sentence reading for "Everyone shot arrows". That reading would seem to be contradicted by the phrase "except Govinda".

It is tempting to want to take the phrase "except Govinda" as a modifier of the sentence "Everyone shot arrows", but to do so would not yield the natural predicate calculus reading of (48) on a compositional semantics, and I have not thought out how such a reading might be attained. At any rate, to make "except Govinda" a sentence modifier is to subscribe to Bhartrhari's theory of a "special rule", for it would then be natural to take (45) as a modifier of (44).

It should be noted that, even allowing Bhartṛhari's position to be correct, the ākāṅkṣā definition of sentences has not been shown to be wrong, since the category of an exception expression will be S/S rather than S, as the Mīmāṃsaka theorist of (43) supposes. At worst it will be the Mīmāṃsaka application of the ākāṅkṣā idea which is discarded rather than the ākāṅkṣā notion itself.

The effects of context. The fragment in Chapter II is intended to be a model of Bhartṛhari's theory of sequences of words, but there are aspects of that theory which it does not model. Bhartṛhari had several things to say about the effects of context on the meaning of sequences of words. He cites two traditional lists of contextual factors in this passage:

(49) (The factors which help to determine the meaning of a word are now discussed):--

> The meanings of words are determined from (their) syntactical connection (in the sentence), situation-context, the meaning of another word, propriety, place, and time, and not from their mere form.
>
> (Another list): (Constant) association (of two things), (their) dissociation, company, and hostility, the meaning (of another word), situation-context, evidence from another sentence, and the proximity of another word.
>
> Words, which, according to their application in one way or another are either nouns or verbs though of identical form, do not have the meaning which they are to convey understood from their form alone (but also from context etc.) (VP II 314-15, 317 Pillai)

Some of these factors seem to me to represent evidence which a competent listener or reader would use to select the correct reading of a sentence from the set of possibilities. Examples are "the meaning of another word, propriety, place, and time" and "the proximity

of another word". Such factors might be incorporated into a pragmatic operator or a set of such operators which select from the set of possible readings of a SLS sentence the most pragmatically appropriate meaning or meanings. The specification of such operators would be difficult and would go beyond the texts since, aside from giving some examples, Bhartṛhari does not discuss them in any detail. It seems enough to have shown how the set of possible readings could be generated here, and to leave the selection of appropriate readings from that set for future research.

There are two other effects of context discussed by Bhartṛhari however which must be noted. One is that in some situations a sequence of words which has expectancy may express a complete meaning. The words "The door", when uttered by a king to courtiers in a drafty room, may mean what these sentences mean: "The door is open" and "Close the door". The courtiers derive the complete meaning from the expectant sequence with the aid of context. Bhartṛhari says that in such situations in which a connection with other meaning is derived from the expectant sequence so that a complete meaning is derived, the words with which the listener began are a sentence:

> (50) When a mere statement of a thing is made (in a sentence) and there is something connected with it, (even then) the sentence is complete verbally without that (something) being stated. (VP II 446, Pillai)

To add this theory to the fragment would involve making the property of being a sentence a two-place relation between sequences of words and situations. Such a definition would complicate the grammar without illuminating the other points to be discussed.

There is another effect of context mentioned by Bhartṛhari which I have not treated. Sometimes the context in which a sentence is uttered is such that the meaning conveyed is not a member of the set of meanings which the grammar of Chapter II assigns to it. Consider this passage:

(51) 'We must go now. Look at the sun'--when time is indicated by implication in this way the idea 'know the time' is conveyed through its means.

A boy who is instructed to protect clarified butter from crows does not prevent himself from protecting it from dogs and the like, the (instructing) sentence having the significance of protecting (the clarified butter), in general. (VP II 310 and 312, Pillai)

That the effect of context may make the meaning of a sentence quite different from its literal meaning is also not treated here. What the grammar in Chapter II is intended to model is the process of deriving what Bhartṛhari calls "primary meanings" from sequences of words. The distinction between primary and secondary meanings is described by Bhartṛhari as follows:

(52) When on merely listening, one understands the word as having a (certain) meaning, that meaning is considered to be principal, and the meaning is secondary where it has to be explained. (VP II 278, Pillai)

Although contextual factors may lead to the derivation of secondary meanings for words, Bhartṛhari believes the process to depend on the derivation of primary meanings:

(53) When a word (in conveying a secondary meaning) depends on itself as functioning in its own meaning (i.e. the principal meaning), then the principal meaning acts as the basis (for the secondary meaning) and the secondary meaning is based on it. (VP II 267, Pillai)

My goal in working out the grammar in Chapter II has been to provide a model for the processing of primary meaning. A model deriving secondary meanings will require something along the lines of the grammar I have given, if (53) is correct, and I believe the grammar of Chapter II to be well fitted to ground extensions which treat secondary meaning.

FOOTNOTES

1 The grammar described in this chapter and formalized in the next was inspired by talks with the linguist Rick Saenz. It was our conjecture that English could be treated as a language in which inflection is determined by position, so that instead of "transformations" of a basic word order there are alternate ways to inflect positionally. The grammar for Sanskrit was first attempted to determine what sorts of formal machinery would be necessary to work this idea out.

A group of University of Massachusetts linguists (and the philosopher James Waldo) are presently investigating this idea.

In developing this system I have been aided and stimulated greatly by talks with Barbara Partee, Terence Parsons, Rick Saenz, James Waldo, Emmon Bach, and other linguists and philosophers at the University of Massachusetts.

2 Note that, since SLS words do not undergo sandhi, the nominative singular of Rāma is Rāmas rather than Rāmaḥ, and so forth.

3 In PTQ Montague refers to the categorial grammar of Ajdukiewicz in Jezyk i Poznanie (Language and Knowledge) as providing some basis for the categorial grammar he devises.

4 For a clear and careful exposition of PTQ, the interested reader is referred to Partee (1975).

5 The synonymy of active and passive formulations is part of the theory of kāraka relations discussed by Panini and elaborated in Book III of the Vākyapadīya. Staal discusses this theory in Staal (1967) in a readable way. To explain how the kāraka theory is formalized by the notation of the predicate calculus is an attractive enterprise which goes beyond the scope of this work. For the reader who believes active and passive formulations are not synonymous, it is possible to correlate a different predicate with each form of the verb, perhaps relating the two with Meaning Postulates.

6 This way of representing expectancy corresponds in some respects to Frege's theory that sentences are "saturated" and are built up from expressions which are "unsaturated". One respect in which the notion of unsaturatedness of Frege does not correspond to the notion of expectancy of the Mīmāṃsakas is that Frege believed names to be saturated, while in the Mīmāṃsaka theory names have expectancy.

CHAPTER II

A FORMAL TREATMENT OF ĀKĀṄKSĀ

1. Surface Level Sanskrit

This chapter contains a characterization of those sequences of Sanskrit words which lack ākāṅkṣā (expectancy). The fragment of Sanskrit for which the characterization is given is called Surface Level Sanskrit (hereafter, "SLS"). SLS is an idealized version of written Sanskrit in that sequences of SLS words do not undergo the sandhi changes which modify (sometimes drastically) sequences of words in natural Sanskrit. To treat the ambiguities which arise when words are taken out of sandhi relationships would complicate the grammar without adding features of relevance to the topics discussed in this work.

SLS is a fragment of Sanskrit which is like that encountered by a student early on in the study of Sanskrit. The lexicon, grammatical principles, and rules of agreement are deceptively simple; the difficulty lies in interpreting a sequence of words somehow or other so that their meaning is understood. In developing this grammar, curiously enough, my being an inexperienced reader of Sanskrit may have been of some aid in that my translation processes are slow enough to be observed and studied, whereas those of an experienced reader would not be so.

The lexicon of SLS consists of the following sets of words:

Common Nouns. The common nouns go 'cow', nara 'man', nārī 'woman', ghaṭa 'pot', gṛham 'house', pada 'word', vākya 'sentence', jāti 'universal', dravya 'substance', artha 'meaning', narasiṃha 'man-lion', and bhavin 'being' in nominative, accusative, instrumental, dative, genitive, and locative cases are members of the set of common nouns.

Example: Included in this set are the following forms of go: gaus '(nom.), gām (acc.), gavā (inst.), gave (dat.), gos (gen.), and gavi (loc.).

Proper Nouns. The proper nouns Rāma, Govinda, Laksmī, Śrī, and Brahman in the six cases listed above are the members of the set of proper nouns.

Example: In this set are the following forms of Crī: Crīs (nom.), Criyam (acc.), Criyā (inst.), Criye (dat.), Criyās (gen.), and Criyi (loc.).

Pronouns. The set of pronouns includes ta and ya in all three genders and six cases.

Example: In this set are the six case forms of ya in neuter: yad, yad, yena, yasyai, yasya, yasmin. (The first two forms will occur only once in the set. They are listed here to maintain the form-case correlation.)

Intransitive Verbs. The verbs mṛ 'die', dhāv 'run', sthā 'stand' or 'exist', and bhū 'exist' in third person singular present, active and passive, (except no passive for bhū) are the members of this set.

Example: Included are tiṣṭhati 'he stands' and sthīyate 'it is stood by'.

1-place Transitive Verbs. This set includes snih 'love', drç 'see', vac 'express', and as 'be' in third person singular present, active and passive, except that as occurs only in active.

Example: Included are the active and passive forms of snih—snihyati and snihyate.

2-place Transitive Verbs. Included in this set are yaj 'sacrifice' and yam 'give' in third person singular present, active and passive.

Example: The set includes the active and passive forms of yam--yacchati and yamyate.

Adjectives. The adjectives sādhu 'good ', cāru 'beautiful', satya 'real', mithyā 'false', and amithyā 'true' in all six cases and all three genders are the members of this set.

Example: sādhu in neuter contributes the forms sādhu, sādhu, sādhunā, sādhune, sādhunas, sādhuni. (Since the nom. and acc. case forms are the same, there are only five members of the set which are neuter forms of sādhu).

Conjunctions. The set includes the three conjunctions ca 'and', vā, 'or', and yadā...tadā 'if...then'.

Not. This set contains na 'not'.

Every. This set contains sarva 'every' in the three genders and six cases.

Any sequence of words from the SLS lexicon is an SLS sequence. Our task is to determine which SLS sequences are sentences of SLS.

2. Categorial Sanskrit

The expressions of SLS will be correlated with members of a categorial language, Categorial Sanskrit ("CS"). The expectancies of the SLS words are coded in the CS categories. In general, a word of category A/B or A/(B,C) has expectancy for a word of type B or words of types B and C in order to produce an expression of type A. Words of CS are in general n-tuples, with members which represent the gender and case of the SLS words with which they are correlated.

The description of CS which follows is intentionally parallel to Montague's exposition in PTQ.

Categories. Let CN, VP, and S be three objects which are non-identical and are not ordered n-tuples. Then the set of categories of CS is the smallest set X such that (1) CN, VP, and A are in X, and (2) whenever A, B, and C are in X, then A/B and A/(B,C) are in X.

We take CN to be the index of the category of common noun expressions, VP to be the index of the category of verb phrases, and S to be the index of the category of sentences.

The following categories of CS will concern us here:

NP, or the category of noun phrases, is S/VP.

1TV, or the category of one-place transitive verbs, is VP/NP.

2TV, or the category of two-place transitive verbs, is (VP/NP)/NP.

Det, or the category of common noun determiners, is NP/CN.

Adj, or the category of adjectives, is CN/CN.

VPBO, or the category of verb phrase scope binding operators, is VP/(NP,VP).

SBO, or the category of sentence scope binding operators, is S/(NP,S).

CNBO, or the category of common noun scope binding operators, is CN/(NP,CN).

Rel, or the category of relative clause coordinators, is CN/(S,CN).

Neg, or the category of sentence modifiers, is S/S.

The category of sentence conjunctions and disjunctions is S/(S/S).

The category of verb phrase conjunctions and disjunctions is VP/(VP,VP).

The category of noun phrase disjunctions is NP/(NP,NP).

Basic Expressions. In order to succinctly specify the set of basic expressions of each category, we use some conventions. B_A is to refer to the set of basic expressions of category A. An expression containing "G" is to stand for the three expressions obtained by substituting the gender markers "M", "F", and "N" for"G." An expression containing "C" is to stand for the six expressions obtained by substituting the six case markers "Nom", "Acc", "Inst", "Dat", "Gen", and "Loc" for "C". An expression containing "i" is to stand for the expressions obtained by substituting every integer for "i".

Example: "⟨go(z),M,C⟩" abbreviates "⟨go(z),M,Nom⟩, ⟨go(z),M,Acc⟩, ⟨go(z),M,Inst⟩, ⟨go(z),M,Dat⟩, ⟨go(z),M,Gen⟩, ⟨go(z),M,Loc⟩".

The basic expressions of each category are the following.

B_{CN} = {⟨go(z),M,C⟩, ⟨nara(z),M,C⟩, ⟨nārī(z),F,C⟩, ⟨ghaṭa(z),M,C⟩,
⟨gṛham(z),N,C⟩, ⟨pada(z),N,C⟩, ⟨vākya(z),N,C⟩,
⟨jāti(z),F,C⟩, ⟨dravya(z),N,C⟩, ⟨artha(z),M,C⟩,
⟨narasimha(z),M,C⟩, ⟨bhavin(z),G,C⟩}.

B_{NP} = {⟨P(Rāma),M,C⟩, ⟨P(Govinda),M,C⟩, ⟨P(Laksmī),F,C⟩,
⟨P(Crī),F,C⟩, ⟨P(ta_i),G,C⟩, ⟨P(ya_i),G,C⟩}.

B_{VP} = {⟨mṛ Nom,Nom⟩, ⟨dhāv Nom, Nom⟩, ⟨sthā Nom,Nom⟩,
⟨bhū Nom, Nom⟩}.

B_{1TV} = {⟨snih Nom Loc,Nom,Loc⟩, ⟨snih Nom Gen,Nom,Gen⟩,
⟨snih Inst Nom,Nom,Inst⟩, ⟨drç Nom Acc,Nom,Acc⟩,
⟨drç Inst Nom,Nom,Inst⟩, ⟨Nom = Nom,Nom,Nom⟩}.

B_{2TV} = {⟨yaj Nom Acc ^Acc,Nom,Acc,Acc⟩,
⟨yaj Inst Nom ^Acc,Nom,Acc,Inst⟩,
⟨yaj Inst Acc ^Nom,Nom,Acc,Inst⟩,
⟨yam Nom Acc Dat,Nom,Acc,Dat⟩,
⟨yam Inst Acc Nom,Nom,Acc,Inst⟩,
⟨yam Inst Nom Dat,Nom,Dat,Inst⟩}.

B_{Det} = {⟨$(x_i)(CN(x_i)\rightarrow P(x_i))$,G,C⟩, ⟨$(Ex_i)(CN(x_i)\&P(x_i))$,G,C⟩,
⟨$(Ex_i)(y_i)((CN(y_i)\equiv x_i=y_i)\&P(x_i))$,G,C⟩}.

B_{VPBO} = {$VPBO_i$}.

B_{SBO} = {SBO_i}.

BCNBO = {$CNBO_i$}.

B_{Adj} = {⟨sādhu,G,C⟩, ⟨mithyā,G,C⟩, ⟨cāru,G,C⟩, ⟨satya,G,C⟩,
⟨amithyā,G,C⟩}.

B_{Rel} = {Rel_i}.

B_{Neg} = {~}.

$B_{S/(S,S)} = \{\&, \vee, \rightarrow\}$.

$B_{VP/(VP,VP)} = \{\vee, \&\}$.

$B_{NP/(NP,NP)} = \{\vee\}$.

There are other expressions in the categories of CS which are built up from the basic expressions. They are described in the next section.

Correlation Rules. The following rules correlate each word of SLS with a set of sequences of basic CS expressions. Each sequence in the set contains CS expressions whose expectancies are those which a reader of the SLS word might associate with it in reading an SLS sentence. For brevity's sake, several conventions are employed.

For any SLS word w of an SLS sequence s,

C(w) denotes the set of sequences of CS expressions correlated with w,

G is "M" if w is masculine, "F" if w is feminine, "N" if w is neuter,

C is "Nom" if w is in nominative case, "Acc" if in accusative, "Inst" if in instrumental, "Dat" if in dative, "Gen" if in genitive, and "Loc" if in locative,

i is the number of w's place in the sequence s,

j is the number of SLS words in s,

Det is $\{\langle(Ex_{2i+1})(CN(x_{2i+1}\&P(x_{2i+1})),G,C\rangle,$

$\langle(Ex_{2i+1})(y_{2i+1})((CN(y_{2i+1}) \equiv x_{2i+1}=y_{2i+1})\&P(x_{2i+1})),G,C\rangle\}$,

as is $\langle Nom=Nom,Nom,Nom\rangle$, (Note: as is the verb 'to be')

$A_1X...XA_n$ is the Cartesian Product of $A_1...A_n$--the set of all ordered n-tuples $\langle a_1...a_n\rangle$ such that $a_1 \in A_1$, ... $a_n \in A_n$.

$A_1 \underline{X} \cdots \underline{X} A_n$ is the Cartesian Product of $A_1 \cdots A_n$ modified in the following way: each member of the Cartesian Product has every ocurrence of "Ø" deleted from it.

"Ø" denotes the empty set.

A U B is the union of A and B--the set which has as members all and only those members of A or B.

Examples of the set operators:

$[\{3\} \cup \{4\}] \times \{\emptyset,2\} = \{3,4\} \times \{\emptyset,2\} = \{\langle 3,\emptyset\rangle, \langle 3,2\rangle, \langle 4,\emptyset\rangle, \langle 4,2\rangle\}.$

$[\{3\} \cup \{4\}] \underline{X} \{\emptyset,2\} = \{3,4\} \underline{X} \{\emptyset,2\} = \{\{3\}, \langle 3,2\rangle, \{4\}, \langle 4,2\rangle\}.$

"Stem"is to be the stem of w and "R" is to be the root of w.

When a SLS word is ambiguous between several forms, the set of expressions correlated with it is to be the expressions correlated with each of its forms taken as a union.

CR1 Common Nouns. If w is a common noun consisting of a stem with gender and case markings, C(w) is the union of these two sets:

1. $\{\langle \text{stem}(z),G,C\rangle\} \underline{X} \begin{Bmatrix}\emptyset\\ \text{as}\end{Bmatrix} \underline{X} (\text{Det} \cup \emptyset)$

2. $\{\langle \text{stem}(z),G,C\rangle\} \underline{X} \begin{Bmatrix}\emptyset\\ \text{as}\end{Bmatrix} \underline{X} \text{Det} \underline{X} \{\langle ta_{2i},G,C\rangle\} \underline{X} \begin{Bmatrix}SBO_{2i}\\ NPBO_{2i}\\ VPBO_{2i}\end{Bmatrix}$

(Note: if C≠Nom, the term containing as is deleted from the product).

The first member of each product represents the expectancy of the common noun itself. The second represents the possible insertion of as 'be' when the common noun is in nominative case. The third represents the possible insertion of a determiner. The fourth and fifth members of set (2) are the pronoun and binding operator required to give the quantifier varying scope.

CR2 Noun Phrases. Each SLS proper noun or pronoun consists of a stem with gender and case marking. For each proper noun, C(w) is

$$\{\langle P(\text{stem}),G,C\rangle\} \underline{X} \begin{Bmatrix}\emptyset\\ \text{as}\end{Bmatrix}.$$

If w is ya in a gender and case, C(w) is

$$\{\langle P(ya_2),G,C\rangle, \ldots \langle P(ya_{2j}),G,C\rangle\} \underline{X} \begin{Bmatrix}\emptyset\\ \text{as}\end{Bmatrix}.$$

If w is ta in a gender and case, C(w) is the union of these sets:

1. $\{\langle P(ta_2),G,C\rangle \ldots \langle P(ta_{2i-2}),G,C\rangle, \langle P(ta_{2i+2}),G,C\rangle \ldots \langle P(ta_{2j}),G,C\rangle\} \underline{X} \begin{Bmatrix}\emptyset\\ \text{as}\end{Bmatrix}$

2. $\{\langle P(ta_{2i}),G,C\rangle\} \underline{X} \begin{Bmatrix}\emptyset\\ NPBO_{2i}\\ VPBO_{2i}\\ SBO_{2i}\end{Bmatrix} \underline{X} \begin{Bmatrix}\emptyset\\ \text{as}\end{Bmatrix}$

3. $\{\langle rel_{2i},G,C\rangle\}$

(Note: if C≠Nom, the term containing as is deleted from the product.)

The first set of expressions correlated with ta represents the appearance of ta as a pronoun to be bound by another binding operator. The second represents with its own binding operator as well as (when "∅" is chosen rather than a binding operator) the possibility of a deictic use of the pronoun. The third represents the possibility that ta may be a relativizing word rather than a pronoun.

CR3 Intransitive Verbs. If w is an intransitive verb with root R,

if w is active, C(w) is $\{\langle R\ \text{Nom},\text{Nom}\rangle\}$.

if w is passive, C(w) is $\{\langle R\ \text{Inst},\text{Inst}\rangle\}$.

CR4 One-place Transitive Verbs. If w is a one-place transitive verb and R is the root of w,

if w is active,

w is snihyati and C(w) is {⟨snih Nom Gen,Nom,Gen⟩, ⟨snih Nom Loc,Nom,Loc⟩}, or

w is pacyati or vacati and C(w) is {⟨R Nom Acc,Nom,Acc⟩}, or

w is asti and C(w) is {⟨Nom = Nom,Nom,Nom⟩}.

if w is passive, C(w) is {⟨R Inst Nom,Nom,Inst⟩}.

CR5 Two-place Transitive Verbs. If w is a two-place transitive verb,

if w is yajati C(w) is {⟨yaj Nom Acc ^Acc,Nom,Acc,Acc⟩}.

if w is yajyate C(w) is {⟨yaj Inst Nom ^Acc,Nom,Acc,Inst⟩, ⟨yaj Inst Acc ^Nom,Nom,Acc,Inst⟩}.

if w is yacchati C(w) is {⟨yam Nom Acc Dat,Nom,Acc,Dat⟩}.

If w is yamyate C(w) is {⟨yam Inst Acc Nom,Nom,Acc,Inst⟩, ⟨yam Inst Nom Dat,Nom,Dat,Inst⟩}.

CR6 Adjectives. If w is an adjective with stem and gender and case markings, C(w) is {⟨stem,G,C⟩}.

CR7 Conjunctions, Disjunctions, Implications. If w is ca then C(w) is {&,&}, where the two &'s are members of $B_{S/(S,S)}$ and $B_{VP/(VP,VP)}$. If w is va then C(w) is {∨,∨, ∨} where the three ∨'s are members of $B_{S/(S,S)}$, $B_{NP/(NP,NP)}$, and $B_{VP/(VP,VP)}$. If w is yada...tada then C(w) is {→}.

CR8. C(na) is {~}.

CR9. If w is sarva in a gender and case, C(w) is the union of the following sets:

1. $\{\langle (x_{2i+1})(CN(x_{2i+1})\rightarrow P(x_{2i+1})),G,C\rangle\}$
2. $\{\langle (x_{2i+1})(CN(x_{2i+1})\rightarrow P(x_{2i+1}),G,C\rangle\} \underline{X} \{\langle ta_{2i},G,C\rangle\} \underline{X} \begin{Bmatrix} VPBO_{2i} \\ NPBO_{2i} \\ SBO_{2i} \end{Bmatrix}$

Disambiguation. To define the sequences of CS expressions associated with a sequence of SLS expressions, we use the operator "$\underline{U}$", defined as follows:

$A_1 \underline{U} \ldots \underline{U} A_n$ is the set of tuples whose first members are the members of a member of A_1, whose next members are the members of a member of A_2, ... whose last members are the members of a member of A_n.

Example: $\{\langle 3,4\rangle, \{2\}\} \underline{U} \{\langle 2,4\rangle\} = \{\langle 3,4,2,4\rangle, \langle 2,2,4\rangle\}$.

The set of disambiguated sequences correlated with any SLS sequence of the form $\langle w_1 \ldots w_n\rangle$ is $C(w_1) \underline{U} C(w_2) \underline{U} \ldots \underline{U} C(w_n)$. Examples of sets of disambiguated sequences are found in the last section.

Combining CS expressions to make expressions of type S. In order to determine which disambiguated sequences of CS expressions yield expressions of type S when expectancy is satisfied, rules are given for combining CS expressions in a categorial way.

A CS expression of type A and a CS expression of type B/A combine to produce a CS expression of type B. CS expressions of types A,B, and C/(A,B) combine to produce a CS expression of type C. The following rules describe the CS expressions which are the result of such combinations, given the following conventions.

"CE(a,b)" and "CE(a,b,c)" is read as "the result of combining expressions a and b" and "the result of combining expressions a, b, and c".

The following functions are used in several of the rules:

F1(p,q,r) is p with every expression of the form P(m) in p replaced by q*, where q* is q with the first occurrence of r in q replaced by m.

Example: F1(P(Rāma), Nom = Nom, Nom) is Rāma = Nom.

F2(p,q,r) is p with every expression of the form P(m) in p replaced by q', where q' is q with every occurrence of r in q replaced by m.

Example: F2(P(Rāma) & dhāv Crī, dhāv Nom, Nom) is dhāv Rāma & dhāv Crī. (q' is dhāv Rāma).

Let G be the gender associated with p. Then

F3(p,q,r) is p with every expression in p of the form P(m) replaced by q*, where q* is q with the first occurrence of r in q replaced by $\begin{Bmatrix} \langle ta_i, G\rangle \\ \langle ya_i, G\rangle \end{Bmatrix}$ if p is $\begin{Bmatrix} P(ta_i) \\ P(ya_i) \end{Bmatrix}$.

F4(p,q,r) is p with every expression in p of the form P(m) replaced by q', where q' is q with every occurrence of r in q replaced by $\begin{Bmatrix}\langle ta_i,G\rangle\\ \langle ya_i,G\rangle\end{Bmatrix}$ if p is $\begin{Bmatrix}\langle P(ta_i)\rangle\\ \langle P(ya_i)\rangle\end{Bmatrix}$.

Example: Suppose ta_4 has gender N. Then F4(P(ta_4), dhāv Nom & mṛ Nom, Nom) is dhāv $\langle ta_4,N\rangle$ & mṛ $\langle ta_4,N\rangle$.

The rules of combination are as follows.

RC1. If a∈Cat CN, b∈Cat NP/CN, a is of the form ⟨s,Ga,Ca⟩, b is of the form ⟨det,Gb,Cb⟩, Ga=Gb, and Ca=Cb, then CE(a,b) is ⟨det',Ga,Ca⟩, where det' is det with every expression of the form $CN(x_i)$ in det replaced by s', where s' is s with every occurrence of "z" in s replaced by x_i.

RC2. If a∈Cat NP, b∈Cat VP, a is of the form ⟨R,G,Ca⟩ and b is of the form ⟨V, Cb⟩ and Ca=Cb, then CE(a,b) is F2(R,V,Ca) unless R is of the form $P(ta_i)$ or $P(ya_i)$. Then CE(a,b) is F4(R,V,Ca).

RC3. If a∈Cat NP, b∈Cat VP/NP, a is of the form ⟨R,Ca,Ca⟩, b is of the form $\langle V,Cb_1,Cb_2\rangle$, and Ca=Cb2, then CE(a,b) is $\langle F1(R,V,Cb_2),Cb_1\rangle$ unless R is of the form $P(ta_i)$ or $P(ya_i)$. Then CE(a,b) is $\langle F3(R,V,Cb_2),Cb_1\rangle$.

RC4. If a∈Cat NP, b∈Cat (VP/NP)/NP, a is of the form ⟨R,Ga,Ca⟩, b is of the form $\langle V,Cb_1,Cb_2,Cb_3\rangle$, and $Ca=Cb_3$, then CE(a,b) is $\langle F1(R,V,Cb_3),Cb_1,Cb_2\rangle$ unless R is of the form $P(ta_i)$ or $P(ya_i)$. Then CE(a,b) is $\langle F3(R,V,Cb_3),Cb_1,Cb_2\rangle$.

RC5. If a∈Cat CN, b∈Cat CN/CN, a is of the form ⟨Ra,Ga,Ca⟩, b is of the form ⟨Rb,Gb,Cb⟩, Ga=Gb, and Ca=Cb, then CE(a,b) is ⟨Ra*,Ga,Ca⟩, where Ra* is Ra with the first expression of the form stem(z) in Ra replaced by an expression of the form Rb(stem)(z).

RC6. If aεCat NP, bεCat VP, c CatεVP/(NP,VP), a is of the form $\langle R,Ga,Ca\rangle$, R is not a pronoun, b is of the form $\langle V,Cb\rangle$, c is of the form $VPBO_i$, V contains at least one occurrence of $\langle ta_i,Ga\rangle$, and for every occurrence of an expression of the form $\langle ta_i,Ga\rangle$ in V Gb=Ga, then CE(a,b,c) is $F2(R,V,\langle ta_i,Ga\rangle),Cb$.

RC7. If aεCat NP, bεCat S, cεCat S/(NP,X), a is of the form $\langle R,Ga,Ca\rangle$, c is of the form SBO_i, b contains at least one occurrence of an expression of the form $\langle ta_i,Ga\rangle$, R is not a pronoun, and every occurrence of an expression of the form $\langle ta_i,Gb\rangle$ in b has Ga=Ga, then CE(a,b,c) is $F2(R,b,\langle ta_i,Ga\rangle)$.

RC8. If aεCat NP, bεCat CN, cεCat CN/(NP,CN), a is of the form $\langle Ra,Ga,Ca\rangle$, b is of the form $\langle Rb,Gb,Cb\rangle$, c is of the form $CNBO_i$, Ra is not a pronoun, Ga=Gb, Ca=Cb, Rb contains at least one occurrence of an expression of the form $\langle ta_i,Ga\rangle$, and for every occurrence of an expression of the form $\langle ta_i,Gc\rangle$ in Rb Ga=Gc, then CE(a,b,c) is $\langle F2(Ra,Rb,\langle ta_i,Ga\rangle),Gb,Cb\rangle$.

RC9. If aεCat S, bεCat CN, cεCat CN/(S,CN), b is of the form $\langle R,Gb,Cb\rangle$, c is of the form $\langle rel_i,Gc,Cc\rangle$, Gb=Gc, Cb=Cc, there is at least one occurrence of an expression of the form $\langle ya_i,Gb\rangle$ in a, and for each occurrence of an expression of the form $\langle ya_i,Ga\rangle$ in a Ga=Gb, then CE(a,b,c) is $\langle b\&a',Gb,Cb\rangle$, where a' is a with every expression of the form $\langle ya_i,Ga\rangle$ in a replaced by "(z)".

RC10. If aεCat S and bεCat Neg, CE(a,b) is b(a).

RC11. If aεCat S, bεCat S, cεCat S/(S,S) then CE(a,b,c) is (a)c(b).

RC12. If a∈Cat VP, b∈Cat VP, c∈Cat VP/(VP,VP), a is of the form ⟨Va,Ca⟩, b is of the form ⟨Vb,Cb⟩, and Ca=Cb, then CE(a,b,c) is ⟨(Va)c(Vb),Ca⟩.

RC13. If a∈Cat NP, b∈Cat NP, c∈Cat NP/(NP,NP), a is of the form ⟨Ra,Ga,Ca⟩, b is of the form ⟨Rb,Gb,Cb⟩, and Ca=Cb, then CE(a,b,c) is ⟨(Ra)c(Rb),G*,Ca⟩, where G* is Ga if Ga=Gb and G* is N if Ga≠Gb. (This is a slight simplification of the actual Sanskrit conditions of subject-predicate case agreement.)

Now we may precisely characterize the sentences of SLS. A SLS sequence s is a SLS sentence if and only if there is a member m of the set of disambiguated sequences for s such that some sequence of application of the Rules of Combination to m transforms m to a single CS expression of type S which contains no expression of the form $\langle ya_i, G\rangle$.

3. Examples

The following examples illustrate the process through which it is determined whether a SLS sequence is a sentence; that is, in these examples we follow the correlation and combination process through to the derivation of an expression of type S.

Example 1 (Common Noun Disambiguation).

s = ⟨Çrīs, gṛham, paçyati⟩

(Çrī in nominative, gṛham 'house' in neuter nominative or accusative, drç 'see' in active).

Possible translations: 'Çrī sees a house' and 'Çrī sees the house'.

Correlated expressions:

By CR2, C(Çrīs)= {⟨P(ÇrĪ),F,Nom⟩, ⟨⟨P(Cri),F,Nom⟩, as⟩}

By CR1, C(gṛham) is the union of these sets:

1. $\{\langle gṛham(z),N,Nom\rangle\}$ X $\begin{Bmatrix}\emptyset\\ as\end{Bmatrix}$ X (Det U$\{\emptyset\}$)

2. $\{\langle gṛham(z),N,Nom\rangle\}$ X $\begin{Bmatrix}\emptyset\\ as\end{Bmatrix}$ X Det X $\{\langle ta_4,N,Nom\rangle\}$ X $\begin{Bmatrix}CNBO_4\\ VPBO_4\\ SBO_4\end{Bmatrix}$

3. $\{\langle gṛham(z),N,Acc\rangle\}$ X (Det U$\{\emptyset\}$)

4. $\{\langle gṛham(z),N,Acc\rangle\}$ X Det X$\{\langle P(ta_4),N,Acc\rangle\}$ X $\begin{Bmatrix}CNBO_4\\ VPBO_4\\ SBO_4\end{Bmatrix}$

(The first two are correlated with gṛham as nominative and the second two with gṛham as accusative.)

By CR4 C(paçyati) = $\{\langle$drç Nom Acc,Nom,Acc$\rangle\}$.

The set of disambiguated sequences correlated with s contains these representative members:

(1) $\langle\langle P(Çr\bar{\imath}),F,Nom\rangle, \langle Nom = Nom,Nom,Nom\rangle, \langle gṛham(z),N,Nom\rangle, \langle$drç Nom Acc,Nom,Acc$\rangle\rangle$.

(This is the only sequence we consider in which as is supplied. The others will fail to yield a sentence for the same reason.)

(2) $\langle\langle P(Çr\bar{\imath}),F,Nom\rangle, \langle gṛham(z),N,Nom\rangle, \langle$drç Nom Acc,Nom,Acc$\rangle\rangle$

(This is the only other sequence we consider in which gṛham is in nominative case. The others will fail to yield a sentence for the same reason.)

(3) $\langle\langle Çr\bar{\imath},F,Nom\rangle, \langle gṛham(z),N,Acc\rangle, \langle$drç Nom Acc,Nom,Acc$\rangle\rangle$

(This is the only sequence we consider in which gṛham is in accusative and has no determiner. The others will fail to yield a sentence for the same reason.)

(4) $\langle\langle Çr\bar{\imath},F,Nom\rangle, \langle gṛham,N,Acc\rangle, \langle(Ex_5)(CN(x_5)\&P(x_5)),N,Acc\rangle,$
$\langle$drç Nom Acc,Nom,Acc$\rangle\rangle$.

(Sequences with the other determiner will produce a sentence in a parallel way).

(5) ⟨⟨P(çrī),F,Nom⟩, ⟨gṛham,N,Acc⟩, ⟨(Ex$_5$)(CN(x$_5$)&P(x$_5$)),N,Acc⟩, ⟨P(ta$_4$),N,Acc⟩, SBO$_4$, ⟨drç Nom Acc,Nom,Acc⟩⟩.

(Sequences with the other binding operators and the other determiner will produce a sentence in a parallel way.)

Sequence (1) fails to produce an expression of type S through combination of its members because after the one-place transitive verb Nom = Nom combines with çrī to produce the intransitive verb çrī = Nom, there is no other nominative phrase to satisfy the expectancy of this verb.

Sequence (2) fails to produce an expression of type S because the verb drç Nom Acc requires an accusative noun phrase and there is none.

Sequence (3) fails to produce an expression of type S for the same reason.

Sequence (4) combines to produce an expression of type S in these steps:

(6) ⟨⟨P(çrī),F,Nom⟩, ⟨(Ex$_5$)(gṛham(x$_5$)&P(x$_5$)),N,Acc⟩, ⟨drç Nom Acc,Nom,Acc⟩⟩ by RC1

(7) ⟨⟨P(çrī),F,Nom⟩, ⟨(Ex$_5$)(gṛham(x$_5$)&drç Nom x$_5$,Nom⟩⟩ by RC3

(7a) (Ex$_5$)(gṛham(x$_5$)&drç çrī x$_5$) by RC2

We may read (7a) as 'there is a thing which is a house and çrī sees it.'

Sequence (5) combines to produce an expression of type S in this way:

(8) $\langle\langle$ Çrī, F, Nom$\rangle$, $\langle(Ex_5)(g\dot{r}ham(x_5)\&P(x_5)),N,Acc\rangle$, SBO_4, $\langle dr\c{c}$ Nom $\langle ta_4,N\rangle\rangle$ by RC1 and RC3

(9) $\langle\langle(Ex_5)(g\dot{r}ham(x_5)\&P(x_5)),N,Acc\rangle$, $dr\c{c}$ Çrī $\langle ta_4,N\rangle\rangle$, SBO_4 by RC2

(10) $(Ex_5)(g\dot{r}ham(x_5)\&dr\c{c}$ Çrī $x_5)$ by RC7

We see that (10) is identical to (7). In this case, addition of binding operators for scope has not changed the reading of the sentence.

Example 2 (Quantification).

s = ⟨sarvas, naras, nārī, snihyati⟩

('every' in masculine nominative, 'man' in masculine nominative, 'woman' in feminine locative, 'love' in active)

Intended readings: 'Every man loves a woman', with two scopes for 'a woman'. (The process is parallel for readings of 'Every man loves the woman.')

By CR9, $C(\underline{sarvas}) = \{\langle(x_3)(CN(x_3)\rightarrow P(x_3),M,Nom\rangle\}$ U

$\{\langle(x_3)(CN(x_3)\rightarrow P(x_3),M,Nom\rangle\}$ X $\{\langle P(ta_2),M,Nom\rangle\}$ X $\begin{Bmatrix} VPBO_2 \\ NPBO_2 \\ SBO_2 \end{Bmatrix}$

By CR1, $C(\underline{naras}) = \left[\langle nara(z),M,Nom\rangle \text{ X } \begin{Bmatrix} \emptyset \\ as \end{Bmatrix} \text{ X } (Det \cup \{\emptyset\})\right]$ U

$\left[\{\langle nara(z),M,Nom\rangle\} \text{ X } \begin{Bmatrix} \emptyset \\ as \end{Bmatrix} \text{ X Det X } \{\langle P(ta_4),M,Nom\rangle\} \text{ X } \begin{Bmatrix} VPBO_4 \\ NPBO_4 \\ SBO_4 \end{Bmatrix}\right]$

By CR1, $C(\underline{nārī}) = \left[\{\langle nārī(z),F,Loc\rangle\} \text{ X } (Det \cup \{\emptyset\})\right]$ U

$\{\langle nārī(z),F,Loc\rangle\}$ X Det X $\{\langle P(ta_6),F,Loc\rangle\}$ X $\begin{Bmatrix} NPBO_4 \\ VPBO_4 \\ SBO_4 \end{Bmatrix}$

By CR4, $C(\underline{snihyati}) = \{\langle snih \text{ Nom Gen},Nom,Gen\rangle, \langle snih \text{ Nom Loc},Nom,Loc\rangle\}$.

The set of disambiguated sequences associated with s contains 1,296 members, including these two:

(11) $\langle\langle(x_3)(CN(x_3)\rightarrow P(x_3)),M,Nom\rangle, \langle P(ta_2),M,Nom\rangle, SBO_2,$ $\langle nara(z),M,Nom\rangle, \langle\bar{n}\bar{a}r\bar{i}(z),F,Loc\rangle, \langle(Ex_7)((CN(x_7)\&P(x_7)),F,Loc\rangle,$ $\langle P(ta_6),F,Loc\rangle, VPBO_6, \langle snih\ Nom\ Loc,Nom,Loc\rangle\rangle$

(12) $\langle\langle(x_3)((CN(x_3)\rightarrow P(x_3)),M,Nom\rangle, \langle P(ta_2),M,Nom\rangle, SBO_2,$ $\langle nara(z),M,Nom\rangle, \langle\bar{n}\bar{a}r\bar{i}(z),F,Loc\rangle, \langle(Ex_7)(Cn(x_7)\&P(x_7)),F,Loc\rangle,$ $\langle P(ta_6),F,Loc\rangle, SBO_6, \langle snih\ Nom\ Loc,Nom,Loc\rangle\rangle$

The members of (11) combine in these steps to produce an expression of type S:

(13) $\langle\langle(x_3)(nara(x_3)\rightarrow P(x_3)),M,Nom\rangle, \langle P(ta_2),M,Nom\rangle, SBO_2,$ $\langle(Ex_7)(\bar{n}\bar{a}r\bar{i}(x_7)\&P(x_7)),F,Loc\rangle, \langle P(ta_6),F,Loc\rangle, VPBO_6,$ $\langle snih\ Nom\ Loc,Nom,Loc\rangle\rangle$ by RC1

(14) $\langle\langle(x_3)(nara(x_3)\rightarrow P(x_3)),M,Nom\rangle, \langle P(ta_2),M,Nom\rangle, SBO_2,$ $\langle(Ex_7)(\bar{n}\bar{a}r\bar{i}(x_7)\&P(x_7)),F,Loc\rangle, VPBO_6, \langle snih\ Nom\ \langle ta_6,F\rangle,Nom\rangle\rangle$

by RC3

(15) $\langle\langle(x_3)(nara(x_3)\rightarrow P(x_3)),M,Nom\rangle, \langle P(ta_2),M,Nom\rangle, SBO_2,$ $\langle(Ex_7)(\bar{n}\bar{a}r\bar{i}(x_7)\&snih\ Nom\ x_7,Nom\rangle\rangle$ by RC6

(16) $\langle(x_3)(nara(x_3)\rightarrow P(x_3)),M,Nom\rangle, SBO_2,$ $\langle(Ex_7)(\bar{n}\bar{a}r\bar{i}(x_7)\&snih\ \langle ta_2,M\rangle\ x_7\rangle$ by RC2

(17) $(x_3)(nara(x_3)\rightarrow(Ex_7)(\bar{n}\bar{a}r\bar{i}(x_7)\&snih\ x_3\ x_7))$ by RC7

We may read (17) as 'For every man there is a woman whom he loves.'

The members of (12) combine with RC1, RC3 and RC2 to produce this sequence:

(18) $\langle\langle(x_3)(nara(x_3)\rightarrow P(x_3)),M,Nom\rangle, SBO_2, \langle(Ex_7)(\bar{n}\bar{a}r\bar{i}(x_7)\&P(x_7)),F,Loc\rangle,$ $SBO_6, snih\ \langle ta_2,M\rangle\langle ta_6,F\rangle\rangle$

The binding operators in (18) may be applied in either order to yield an expression of type S. If SBO_6 is applied before SBO_2, (17) is derived. If SBO_2 is applied before SBO_6, this expression is derived:

(19) $(Ex_7)(n\bar{a}r\bar{i}(x_7)\&(x_3)(nara(x_3)\rightarrow snih\ x_3\ x_7))$ by RC7

We may read (19) as 'There is a woman who is such that every man loves her.'

Example 3 (Relative Clauses).

s = ⟨Rāmas, tam, gām, Çrīyā, yamyate, yas, dhāvati⟩

('Rāma' in masculine nominative, 'such that' in masculine accusative, 'cow' in masculine nominative, 'Çrī' in feminine instrumental, 'give' in passive, 'which' in masculine nominative, 'run' in active.)

Literal reading: 'Rāma is given a cow by Çrī which runs'.

Intended reading: 'Çrī gives Rāma a cow which runs'.

The relevant members of the sets of correlated expressions are these:

By CR2, C(Rāmas) includes ⟨P(Rāma),M,Nom⟩.

By CR2, C(tam) includes $\langle rel_4, M, Acc\rangle$.

By CR1, C(gām) includes ⟨⟨go(z),M,Acc⟩, ⟨(Ex7)(CN(x7)&P(x7)),M,Acc⟩, $\langle P(ta_6), M, Acc\rangle$, SBO_6⟩.

By CR2, C(Çrīyā) includes ⟨P(Çrī),F,Inst⟩.

By CR5, C(yamyate) includes ⟨yam Inst Acc Nom,Nom,Acc,Inst⟩.

By CR2, C(yas) includes $\langle P(ya_4), M, Nom\rangle$.

By CR3, C(dhāv) includes ⟨dhāv Nom,Nom⟩.

When we apply the first steps of combination to these members we obtain

(20) ⟨⟨rel_4,M,Acc⟩, ⟨go(z),M,Acc⟩, ⟨(Ex7)(CN(x7)&P(x7)),M,Acc⟩, SBO_6, yam Çrī ⟨ta_6,M⟩ Rāma, dhāv ⟨ya_4,M⟩⟩ by RC4,RC3 and RC2

(21) $\langle\langle go(z)\&dh\bar{a}v(z), M, Acc\rangle, \langle(Ex_7)(CN(x_7)\&P(x_7)), M, Acc\rangle, SBO_6,$ yam $\acute{S}r\bar{i}$ $\langle ta_6, M\rangle$ Rāma $\rangle$ by RC9

(22) $\langle\langle(Ex_7)((go(x_7)\&dh\bar{a}v(x_7))\&P(x_7)), M, Acc\rangle, SBO_6,$ yam $\acute{S}r\bar{i}$ $\langle ta_6, M\rangle$ Rāma$\rangle$ by RC1

(23) $(Ex_7)((go(x_7)\&dh\bar{a}v(x_7))\&$yam $\acute{S}r\bar{i}$ x_7 Rāma) by RC2

We may read (23) as 'There is a thing which is a cow and runs and $\acute{S}r\bar{i}$ gives it to Rāma'.

4. A SLS Transformational Grammar.

The following Transformational Grammar is strongly equivalent to the categorial grammar for SLS.

Phrase Structure Rules.

PS1 NP → {Det, CN}

PS2 S → {NP, VP}

PS3 VP → {1TV, NP}

PS4 1TV → {2TV, NP}

PS5 CN → {Adj, CN}

PS6 VP → {NP, VPBO, VP}

PS7 S → {NP, SBO, S}

PS8 CN → {NP, CNBO, CN}

PS9 CN → {CN, Rel, S}

PS10 S → {Neg, S}

PS11 S → {S, SConj, S}

PS12 VP → {VP, VPConj, VP}

PS13 NP → {NP, NPConj, NP}

Tree Construction.

A structural tree is a tree t such that

(1) the topmost node of t is S,

(2) for any node n of t which immediately dominates nodes $m_1 \ldots m_n$, there is a PS rule of the form $X \rightarrow \{m_1 \ldots m_m\}$, and

(3) no terminal node of t is S.

From the set of structural trees we derive those trees which are correlated with SLS sentences in this way:

For each structural tree t form the set of all sentence trees based on t by adding a node n' below each terminal node n of t such that

(1) if n is of the form X, $n' \in B_X$,

(2) if n is the ith terminal node of t and n is of the form Rel, SBO, CNBO, or NPBO, then the subscript of the first member of n' is 2i,

(3) if n' is of the form $\langle ta_j, G, C \rangle$ or $\langle ya_j, G, C \rangle$, and there are k terminal nodes on t, then $j \leq 2k$, and

(4) if n is Det and n is the ith terminal node of t then the first member of n is $(x_{2i})(CN(x_{2i}) \rightarrow P(x_{2i}))$ or $(Ex_{2i})(CN(x_{2i}) \& P(x_{2i}))$, or $(Ex_{2i})(y_{2i})((CN(y_{2i}) \equiv x_{2i}=y_{2i}) \& P(x_{2i}))$.

Not all sentence trees based on t will represent SLS sentences, because not all will have constituents which are properly bound and agree in gender and case. Hence we apply the following procedure to each sentence tree t' based on a structural tree t to determine whether t' is a grammatical sentence tree:

(1) For each terminal node n of t',

if $n \in B_{CN}$, B_{NP}, B_{Det}, B_{Adj}, or B_{Rel}, write the gender and case of n to the right of the node which immediately dominates n.

if $n \in B_{IV}$, B_{1TV}, or B_{2TV}, write the last member of n to the right of n', the node which immediately dominates n.

if $n \in B_{1TV}$ or B_{2TV}, write the next-to-last member of n to the right of the node which immediately dominates n'.

if $n \in B_{2TV}$, write the second member of n to the right of the lowest VP node which dominates n.

(2) Each nonterminal node of t' except the highest is sister to one or two nodes formed by a Phrase Structure rule from a node which immediately dominates them. Beginning with the lowest nonterminal nodes and working up, add symbols to t' and test for grammaticality by applying these rules to each node (where "PS(n, $m_1 \ldots m_j$)" means "$m_1 \ldots m_j$ were derived from n through PS_1"):

i. if PS1(n, m_1, m_2) or PS5(n, M_1, m_2), the gender and case of m_1 and m_2 must be the same. If so, write them to the right of n.

ii. if PS2(n, m_1, m_2) or PS3(n, m_1, m_2) or PS4(n, m_1, m_2), the case marking on m_1 and m_2 must be the same.

iii. if PS6(n, m_1, m_2, m_3) or PS7(n, m_1, m_2, m_3) or PS8(n, m_1, m_2, m_3), the gender and case of m_1 must be identical to the gender and case of some terminal node dominated by m_3 of the form $\langle ta_i, G, C \rangle$, where i is the subscript of the node dominated by m_2, and every such node must have the same gender as m_1.

If so and if n is VP, write the case to the right of m_3 to the right of n.

iv. if PS9(n, m_1, m_2, m_3) the gender of m_1 must agree with the gender of some terminal node of the form ⟨va_i,G,C⟩ dominated by m_3, where i is the subscript of the node dominated by m_2, and it must agree with the gender of every such terminal node. If so, write the gender and case of m_1 to the right of n.

v. if PS13(n, m_1, m_2, m_3), write the gender of m_2 and m_3 to the right of n if they are the same. Otherwise, write "N" to the right of n.

vi. if PS6(n, m_1, m_2, m_3) or PS7(n, m_1, m_2, m_3) or PS8(n, m_1, m_2, m_3), write "B" (for "Bound") to the right of every terminal node dominated by m_3 of the form ⟨ta_i,G,C⟩, where i is the subscript of the node dominated by m_2.

vii. if PS9(n, m_1, m_2, m_3), write "B" to the right of every terminal node dominated by m_3 of the form ⟨va_i,G,C⟩, where i is the subscript of the node dominated by m_2.

If each nonterminal node of t' satisfied these conditions and every pronoun of the form ⟨va_i,G,C⟩ has "B" to its right when the process is finished, then t' is a grammatical sentence tree.

There is one optional transformation:

Scrambling.

W X Y Z

SD 1 2 3 4

SC 1 3 2 4

Condition: 3 and 2 are terminal nodes or tadā or yadā.

(Probably the best way to represent structure after application of this transformation is to maintain the lines of dominance, although they may cross if it is widely applied.)

SLS sentences are derived from grammatical sentence trees by the application of this procedure:

i. apply the transformation as many times as desired to the trees.

ii. process the terminal nodes of the trees from left to right in this way:

a. if the nodes are binding operators or relativizers, write nothing;

b. if the nodes are the existential or definite determiners, do not write them;

c. if the nodes are of any other category, write the member of the SLS lexicon associated with them (that is, for $\langle ta_i, N, Acc \rangle$ write tam; for $\langle$yam Nom Acc Dat,Nom,Acc,Dat$\rangle$ write yacchati. Exception: $\langle$Nom = Nom,Nom,Nom$\rangle$ is written optionally;

d. For every occurrence of a binding operator with subscript i, one pronoun of the form $\langle ta_i, G, C \rangle$ is not written.

I have asserted without proof that the transformational grammar and the categorial grammar are strongly equivalent. I shall not prove that result here, producing instead the trees for the example sentences of the previous section and comparing them to trees of the categorial grammar constructed in the following way:

Each sequence of SLS words which is a sentence has a member m of its set of disambiguated sequences which combines to produce an expression of type S with all ya pronouns bound. Create a tree t for each such m in this way:

i. each member of m is a terminal node of t.

ii. each terminal node n of t is immediately dominated by a node of the form X, where $n \in B_X$.

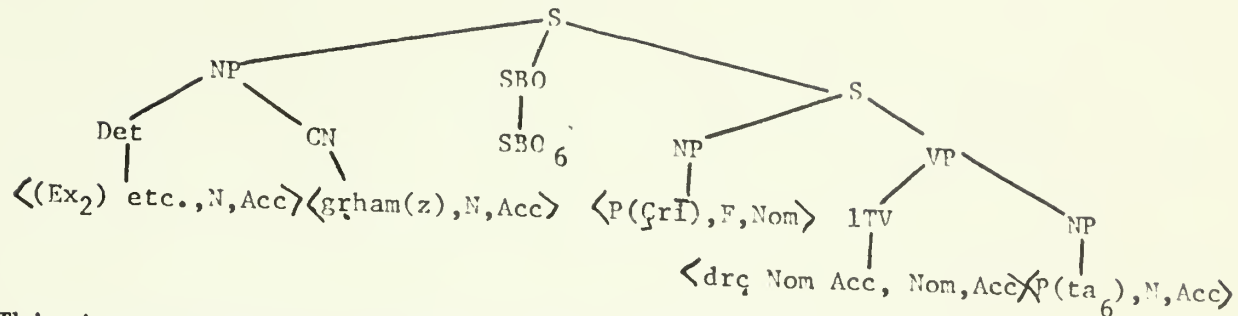
iii. where two or three expressions were combined by a Rule of Combination to produce an expression of category X, write X as the node which immediately dominates the indices of their categories on t.

Any SLS sequence s which is a sentence according to either system has a tree in both systems which is identical except (possibly) for subscripts on its variables, binding operators, and relativizers. This result was suggested by the material in Cooper and Parsons (1976), and some of the techniques used there have been employed here.

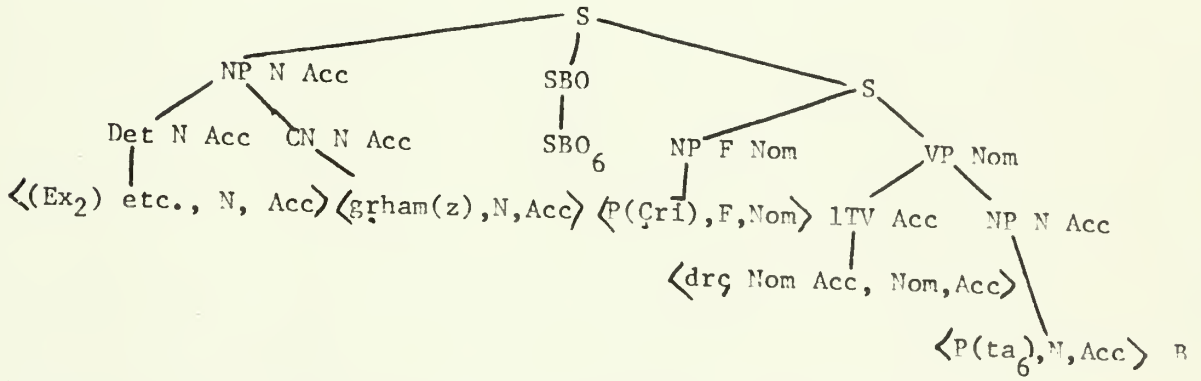
Let us consider some examples.

Example 1 $s = \langle$Cris, grham, paçyati$\rangle$.

The sentence tree for s is



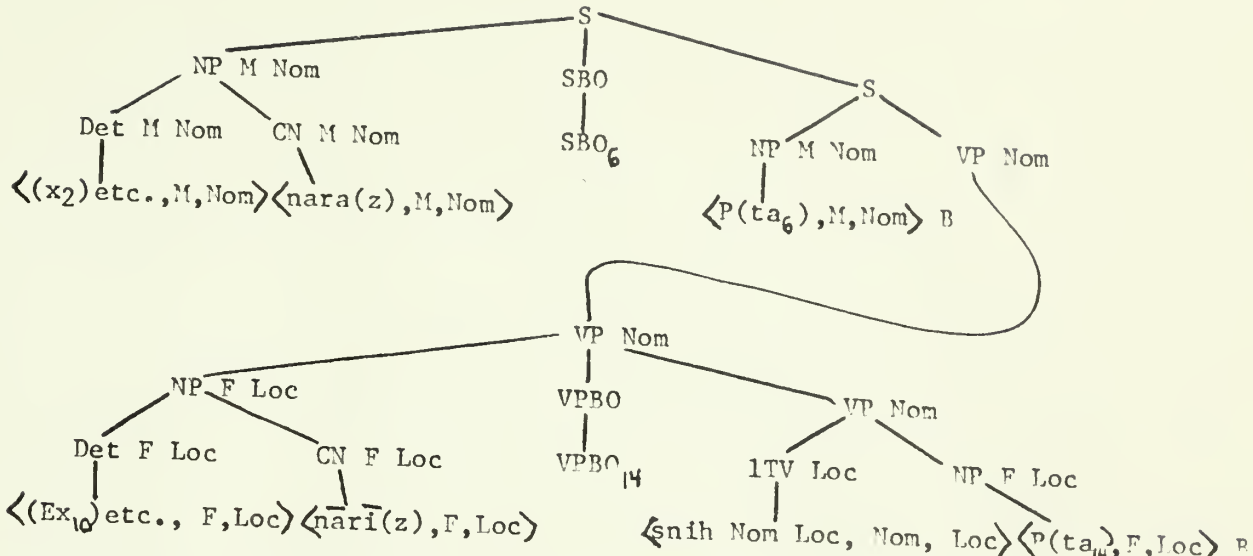
This is a grammatical sentence tree. When processed, it looks like this:



Except for differing subscripts, the tree for example 1 of the previous section is identical to this one.

Example 2 (as in the previous section) s = ⟨sarvas, naras, nāri, snihyati⟩.

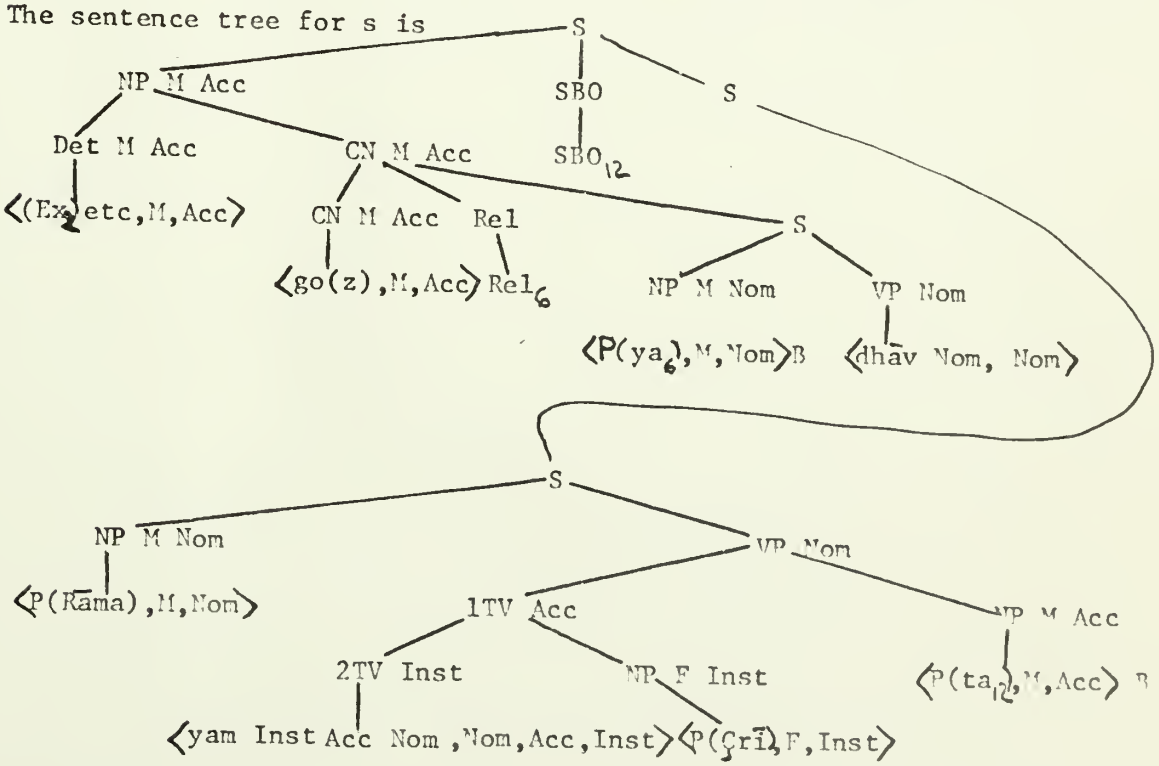
With process markings, the sentence tree for s is the following:



The tree for Example 2 of the previous section, sentence (11) is identical to this one without its process markings, except for variations in the subscripts.

Example 3 s = ⟨Rāmas, tam, gām, Çriyā, yamyate, yas, dhāvati⟩.

The sentence tree for s is



The tree for Example 3 of the last section, reading (23), when rewritten, is identical to this tree except that it has variations in the subscripts.

CHAPTER III

THE SPHOṬA THEORY

1. Introduction

The doctrine of the Grammarians which is best known in the history of Indian philosophy of language is the theory of sphoṭa, a theory which Bhartṛhari is generally acknowledged to have originated in its classical form. Due to a combination of factors, however, scholars are not agreed on the nature of sphoṭa in Bhartṛhari's theory. In this chapter reasons for their disagreement are discussed and the currently accepted view is considered. The fragment developed in Chapter II is used to support the claim that the currently accepted view is incorrect.

2. Grounds for Disagreement

That the sphoṭa theory was influential in the history of Indian philosophy of language is denied by nobody. In fact, Chakravarti writes,

> (1) Though it [the theory of sphoṭa] embodies, so to speak, the crowning achievement of all grammatical speculations... (Chakravarti (1930), p. 111)

That it is Bhartṛhari's version which constitutes the classical formulation of the sphoṭa theory is denied by very few. For example, Raja writes

> (2) The theory of sphoṭa is one of the most important contributions of India to the central problem of semantics in general linguistics....This sphoṭa theory was fully developed and systematized by the great grammarian-philosopher Bhartṛhari in his Vākyapadiya... (Raja (1963), pp. 97-98)

Despite this agreement on the theory's importance, there is an astonishing range of disagreement on the nature of the sphoṭa theory itself. It is best to document this disagreement by quoting some texts which illustrate it.[1]

Sphoṭa as hypostatization of sound. A. Berriedale Keith writes, in A History of Sanskrit Literature,

> (3) the sphoṭa is a mysterious entity, a sort of hypostatization of sound, of which action sounds are manifestations. (p. 387.)

Sphoṭa as sound of word as a whole. S. K. De writes in Studies in the History of Sanskrit Poetics,

> (4) The sphoṭa...may be explained as the sound of a word as a whole, and as conveying a meaning apart from its component letters (varṇas). The sphoṭa does not contain exactly the sounds of the word in the order peculiar to the letters, but the sounds or something corresponding to them are blended indistinguishably into a uniform whole. (p. 180, vol. ii)

Sphoṭa as eternal sound. Cowell translates sphoṭa as "sound" of an eternal sort:

> (5) And...(say the wise in these matters)...this sphoṭa is an eternal sound distinct from the letters and revealed by them, which causes the cognition of the meaning. (translation by E. B. Cowell and Gough of the Sarvadarśana-saṃgraha, p. 211)

In his Sanskrit-English Dictionary, Monier-Williams defines the sphoṭa as "Sound (considered as eternal, indivisible and creative)". (p. 1270).

Sphoṭa as meaning. In the Heyapākṣa of Yoga, P. V. Pathak writes

> (6) One can go to the length of identifying the sphoṭa with the meaning of the word. (p. 84)

Later he writes,

(7) The sphoṭa theory, at its worst, is only a hypostatization of a psychological process of perception.

Sphoṭa as meaning-bearer. The view which is held by the most recent commentators on the Grammarians (and the most careful) is that the sphoṭa is an entity which expresses the meaning. Raja writes

(8) ...the discussions of the grammarians on the sphoṭa theory make it clear that the sphoṭa is not the idea or the meaning, but it is that indivisible symbol which brings to light the idea of the thing-meant. (Raja (1963), p. 145)

Sphoṭa as Brahman. Bhattacharya writes

(9) The ultimate truth according to the Grammarians is Śabda-Brahman, the sphoṭa par excellence... (Bhattacharya (1962), p. 14)

My intention in displaying this diversity of opinion is to demonstrate that the topic is one concerning which a great deal of confusion exists. The fault, however, does not lie with the scholars quoted. Rather, it lies primarily with the texts having to do with sphoṭa. Let us note two difficulties which afflict the scholar of the sphoṭa doctrine.

Variations in the theory. There were many sphoṭa-theorists, and they did not agree. In Patañjali's Mahābhāśya the theory was this:

(10) Thus it is clear that for Patañjali the sphoṭa is a unit of sound as an isolated letter, or a series of letters which can be analysed as a succession of sound-units; it has a normal and fixed size, and is entirely different from the sphoṭa of the later grammarians, which has no size or parts. (Raja (1963), p. 102)

Bhartṛhari wrote a commentary on the Mahābhāśya, and he was familiar with this theory of sphoṭa as well as several others which are mentioned in Vākyapadīya I, 81, 102, and 106.

Bhartṛhari's successors also proposed their versions of the sphoṭa theory, many of which differed from the one I will attribute to him.

The lack of consistency among the scholars reflects, then, the lack of consistency among the writers in the sphoṭa tradition. A writer who takes one form of the theory as the theory of sphoṭa will disagree with a writer who uses another form as the paradigm. There are problems enough in explicating the version found in the Vākyapadīya. Here the goal is simply to do that, and leave the connections to other versions for later work.

Terminology. Another problem which hinders the commentator on Bhartṛhari's theory is the fact that the theory is rarely stated with the use of the word sphoṭa. Most of the passages which are taken to describe sphoṭa use the word śabda or the word vākya, and it is a matter of interpretation when such uses are to be taken to apply to sphoṭas. If one takes a hard line here, it is possible to hold that Bhartṛhari never stated the classical sphoṭa theory. Such a hard line is described by Iyer in this passage:

> (11) [some commentators] go further and say that even Bhartṛhari, who came much later than Patañjali, did not call the meaning-bearing unit sphoṭa. Here is what Dr. S. D. Joshi says--
>
> (1) "Bhartṛhari has nowhere clearly stated in his Vākyapadīya that sphoṭa is over and above the sounds, it is indivisible, and without any inner sequence and it is a meaning-bearing unit of language... (Sphoṭanirṇaya of Kaunda Bhatta, p. 29)
>
> (3) All this confusion has arisen in the mind of later grammarians because they identify Patañjali's conception of śabda with the concept of sphoṭa. (p. 39)

(4) In the context of meaningful speech-unit, Bhartṛhari has never used the term sphoṭa." (p. 40)
(Iyer (1969), p. 57)

The fascinating thing about Joshi's interpretation is that the data on which it is based seems to me to be correct. The interesting characteristics of sphotas in the theory are read in from passages which use śabda or vākya to denote the entities which have those characteristics. Thus, although Bhartṛhari is recognized as the first Grammarian to develop the sphoṭa theory in an interesting way, it is a matter of interpretation just what that theory was, and its ultimate form depends on which passages one reads as passages in which sphoṭa and śabda or sphoṭa and vākya are synonymous. That there is some synonymy between them is argued persuasively by Iyer in passages such as this one:

(12) For instance, Vak. I.77 says that after the manifestation of the word (śabda) the secondary sounds cause difference in the speed of utterance, but the essence of the sphoṭa is not affected by them. Here, obviously, śabda and sphoṭa stand for the same thing and if śabda is the meaning-bearing unit, the sphoṭa is automatically so.
(Iyer (1969, p. 159)

On the basis of such texts, Iyer asserts

(13) While it is true that the meaning-bearing unit is usually referred to as śabda in the Vākyapadīya, it is also true that the words śabda and sphoṭa are used as synonyms, sometimes in the same stanza. (Iyer (1969), p. 159)

A point Iyer does not bring up (and which vitiates his argument, although not fatally) is that there are also passages in which śabda and sphoṭa are clearly used non-synonymously. The following is an example:

(14) Whether the speech-sound (śabda) is short or long, the measure of the Word (sphoṭa) does not change.
(VP I 103, Pillai's translation).

(Pillai also translates VP I 77 as a passage in which śabda and sphoṭa contrast in meaning, but I do not believe he meant to do it.)

I have puzzled over these texts and difficulties for some time and finally came to the following conclusion. It makes no difference whether we follow Joshi and take a hard line or not, for following Joshi we derive a sphoṭa theory under another name; probably it should be termed the śabda theory of Bhartrhari. To follow the tradition of the commentators and the most modern thought on the matter is to derive a similar theory through a wide assumption that śabda and sphoṭa are synonymous--a theory more properly called the sphoṭa theory. Since it is the theory I am interested in and not its correct title, I shall bow to tradition and term the theory to be given here the sphoṭa theory, although it seems to me that Joshi's point is well taken.

Unclear texts. Allowing synonymy between śabda and sphoṭa at appropriate points resolves some of the difficulties of interpretation, but one major problem remains. In the act of communication through speech, there are sounds, there are words, and there are word-meanings. What is the relationship between words and word meanings? In particular, is the relationship one of identity or is it some other very intimate relation? These two theories are contrasted in the Vākyapadīya in the following passages:

> (15) Grammarians consider that there are two 'word-entities' (i.e., two elements) in functional words; one (i.e., the sphoṭa) is the cause of the (production) of words and the other (the speech-sound) is used in connection with meanings.

> Some, among the teachers of old considered that there was a difference in essence between these two. Others (on the other hand) speak of the same undivided entity being thought various, through a difference in conceiving it.
> (VP I 44-45. Parenthesized words Pillai's).

(16) In one grammatical statement the view is held that there is identity between that which expresses a meaning, and the meaning expressed; while in some other places it is stated that there is no such identity. (VP II 98. Pillai)

The difficulty is that Bhartṛhari never states which of the two theories--an identity theory of words and meanings and a non-identity thoery of words and meanings--he prefers, although he writes

(17) Speech and meaning being the two halves of one fact, are not distinct and separable. (VP II 31b, Pillai.)

This passage is not ultimately satisfying, since the two halves of an object are not identical, whereas things which are not distinct are identical. Thus both theories have support in this text.

The confusion on the part of commentators, then, is amply abetted by Bhartṛhari's method of citing theories without always stating which ones he held. In this case he cites both theories, fails to declare himself in favor of either, and writes in different passages as though he held each.

The view which is carefully set forth by Brough, supported by Raja, and endorsed by Iyer, is that sounds express sphoṭas, which express meanings. Raja's statement of this theory is quoted in (8). Iyer's is as follows

(18) Bhartṛhari's chief point is that what is called śabda is not the sounds uttered and heard in a sequence but an entity over and above them and it is that and that alone which can convey the meaning. (Iyer (1969), p. 160)

In this view sphoṭas function as meaning-bearers and are, therefore, not identical to the meanings they bear.

It is a characteristic feature of the non-identity theory that word-sphoṭas are held in mind after sounds are heard and before meanings are in mind when language is processed. Similarly, sentence-sphoṭas come to mind between words and sentence-meanings. While this theory accounts adequately for the data it is intended to account for, I do not believe it accounts for the way all people process language. In order to make this point clearly, let us introduce two new grammars. In one, the grammar VS (for varṇa-sphoṭa 'letter-sphoṭa'), entities which play the part of word-sphoṭas in the non-identity theory come between letters and word-meanings.[2] In the other, the grammar VS', no such entities are employed. I believe both grammars model the way language is processed. (In what follows, I substitute letters or syllable characters for sounds in modelling the identity and non-identity theories. It is simpler to write about characters than sounds in a paper of this sort. The principle is the same for either sort of example.)

The VS Grammar. The grammar of VS has a simple syntax and a more complicated semantics. The syntax is as follows.

Any syllable of the Devanagari system of characters is a character of VS, the space " " is a character of VS, and nothing else is.

Any sequence of VS characters is a VS sequence.

The semantics of VS is as follows. Correlated with each character of VS is a function subject to the following constraints:

(19) a. The name of each function is the English transliteration of the Devanāgari character.

b. No function is a word or a word-meaning,

c. the result of applying a function with name a to a function with name b is a function with a name which is the result of concatenating a and b, and

d. the result of applying a function with name a to the space is
   (i) ∅ if a is not a member of the SLS lexicon
   (ii) a if a is a member of the SLS lexicon.

Examples: Correlated with न (pronounced "na") is the function named na. Correlated with रा (pronounced"rā") is the function named rā. When the function na takes the function rā as argument, the result is the function named narā.

For convenience, let us denote the function correlated with the space as "#". Then we use the following convention. The partition of a sequence of functions is the set of smallest subsequences such that each subsequence terminates in "#" (exception: the last subsequence need not so terminate.)

Example: the sequence ⟨rā,me,ṇa,#,dha⟩ is partitioned into ⟨rā,me,ṇa,#⟩ and {dha} .

A VS word is a VS sequence which contains a single # in final position and which yields a non-empty result when the first member of the sequence is applied to the second and the result is applied to the third and so on until the result is applied to the space. Example: The sequence ⟨rā,me,ṇa,#⟩ is a VS word because the result of applying rā to me (rāme) applied to ṇa (rāmeṇa) applied to "#" (the SLS word Rāmeṇa) is not the empty set, but the word Rāma in instrumental case.

A VS sentence is a VS sequence composed of VS words which have a set of associated sequences which may interact as in the grammar in Chapter II to produce an expression of type S.

The point of introducing this model is to show that there is a way to process language beginning with letters such that the meaning of the sentence is built up from the meaning of the letters and so that the words are built up out of letters in a way parallel to the way sentences are built up out of words. This model seems to me to correspond to the way people process language when they read with the verbal accompaniment "rā...me...na...rāmena!" First the sounds (or characters) are processed, then they are concatenated to form a word when a word-boundary is reached, then the meaning is attached to the word.

While such a model fits this way of processing langauge, it is incorrect as a theory of the way all language processing takes place. To see this, let us construct the grammar VS' which begins with letter meanings as before and derives sentence meanings without going through the intermediate step of deriving word-symbols which express word-meanings.

VS' is like VS except for the following modification. The last requirement on the functions associated with VS' characters is

(20) d. the result of applying a function with name a to the space is
(i) $\emptyset$ if a is not a member of the SLS lexicon
(ii) C(a) if a is a member of the SLS lexicon

In VS' the meaning of a VS word is the set of word-meanings associated with its SLS counterpart rather than the SLS word itself. These word-meanings interact to produce a sentence-meaning as in VS and the result is identical. The only difference is that the process is one step shorter.

It is, then, formally possible to eliminate the word-sphoṭa from one's model of language processing. Is there evidence to the effect that this formalism portrays the processing accurately? I believe there is. The VS' model seems to fit the way of reading which often has this sort of verbal accompaniment: "Rā...mas...dhā... va...ti...Rāma runs!" No impression of the word as a whole need come to mind when one is an experienced reader because one is conditioned to correlate meanings directly with sequences of syllables. For this reason among others it seems to me that the non-identity theory is incorrect as a model of the way all language-processing goes on, although it is correct as a model of some processing--that of a beginner who is slow and easily observed. (It is, of course, open to a non-identity theorist to claim that word-sphoṭas are called to mind by people who read in the VS' way, although no verbalization corresponding to their appearance is produced. While it is possible to claim this, I believe there is no evidence that the claim is correct.)

There is another class of examples of language processing in which it appears that no word-sphoṭas (in the sense of the non-identity theory) are used. A skilled reader often takes in language in segments comprised of several words. A reader who reads in phrases rather than words is such a processor. The sentence "Mary gave a ball to Bill", for example, might be read as composed of these parts: "Mary", "gave a ball", and "to Bill". To read it in such a way is to short-circuit the process of deriving sentence-meanings from sentences in a different way from that of a VS' reader, for such a

reader seems to associate the meaning of "gave a ball" with the three-word phrase directly, rather than building it up word by word (or letter by letter). The words "gave a ball" function as a single name for the meaning when read in this way, and there is therefore no occurrence of the word-sphotas correlated with "gave", "a", and "ball" in the mind of the phrasal reader. (Of course, it is open to a non-identity theorist to claim that the word-sphotas are called to mind but they are overlooked in the speed with which the reader processes. Although one can claim this, I cannot think of any evidence that the claim is true.)

An extreme example of this sort of phenomenon is one which I know of directly. I was trained in French with a method which relied heavily on the memorization of dialogues which were recited and acted out. Later, when I spoke and understood French without great proficiency, I was involved in a conversation in which the question "Voulez-vous allez chez moi?" was asked. I responded immediately, without thought, for this very sentence had occurred in a dialogue I had memorized and acted out months before. My response was based on knowing the meaning of the sentence, and it was not the response of the dialogue, so I had not given an answer which I was conditioned to give. If even a single word of the sentence had been changed (as I discovered upon observing other instances of my processing of French) the sentence would have been as slow and difficult to understand as the others in the conversation, for I did not process French rapidly enough to derive the meaning of the sentence from its words as quickly as I did. My conclusion is that the sounds functioned as a unit

which called the sentence meaning to mind directly, without the intervention of words and word meanings.

If I have described what happened correctly, the non-identity theory of language processing is incorrect as a description of what always occurs when speech is understood, for no sphoṭas seem to have come to mind when I understood the French sentence. The identity theory does provide a correct account of the processing, however, for to understand sounds is to derive their meanings and (on this theory) their meanings are sphoṭas.

Examples of the sort just described seem to be the rule rather than the exception in language processing, but it is primarily in the experienced processor's comprehension of speech that they occur. In such cases the stages in processing are so rapid that they are not easily observed, and it is tempting to think that they must be the same stages which an inexperienced processor goes through. In my case I was fortunate to be inexperienced enough to process slowly, except for the sentences which I had been conditioned to process quickly in another way.

The non-identity theory is incorrect, then, as far as the evidence goes. It seems to have been accepted by Brough, Raja, and Iyer for two reasons: They thought it was correct, and later Grammarians explicitly held it. Believing it to be incorrect, I have been fascinated by the fact that Bhartrhari does not explicitly state it, and have searched for indirect evidence that it is the identity view instead which Bhartrhari had in mind. There are two sources of such evidence.

In (15) Bhartṛhari says that Grammarians consider there to be two "word-entities" in functional words, although he does not say what they are. One causes words to be produced and the other is connected with meanings. Iyer reads this passage as asserting that sphoṭa and meaning together make up the integrated word. Read in this way the passage supports the identity theory. Pillai, on the other hand, reads it in another way. His parenthetical insertions identify the first element with the sphoṭa and the second with speech-sounds. Read in this way the passage is compatible with either theory, for it has no bearing on the question whether sphoṭa and meaning are the same. Read in this way it is not problematic whether or not Bhartṛhari held the identity theory, for it is stated often enough elsewhere that he thought sphoṭas and speech-sounds were different.

In (16) a similar difference in readings follows. If we take "that which expresses a meaning" to be speech-sounds, it is clear that Bhartṛhari holds the non-identity theory, and the older authorities he refers to are Grammarians like Patanjali, who held that sphoṭas have duration and other properties of sounds.

Read in this way, neither of the two passages concerns the controversy over the identity and non-identity theory as it has been stated here. Furthermore, Given Iyer's reading in which the passages do concern this controversy, it is a puzzle why Bhartṛhari never declared himself for either of the two. Given the reading which Pillai endorses for (15) and the reading I prefer for (16), there is no puzzle. Bhartṛhari has said often enough in other sections that the meaning of a word or a sentence (as sounds) is different

from the sounds themselves. Given an identity reading of the passages (15) and (16), then, Bhartṛhari is made to look less coy with respect to a topic on which he should have been quick to pronounce judgment.

A second sort of evidence that Bhartṛhari held the identity theory of sphoṭa and meaning concerns his description of the derivation of sentence-meanings from words. According to the non-identity theory, word-meanings (expressed by word-sphoṭas) would interact to produce a sentence-sphoṭa, which would then express a sentence-meaning. This description is not that given by Bhartṛhari. In a celebrated passage, Bhartṛhari writes:

> (21) When the word-meanings in a sentence are detached (from out of the sentence) and (thus) understood, a different flash of insight is produced (out of it). That (flash of insight) presented by the word-meanings is described as the meaning of the sentence.
>
> It is by no means describable to others in such terms as "it is like this". Having been formed from the function of one's inner self, its nature is not known even to the person.
>
> It effects the fusion of the (individual) word-meanings, without itself being logically thought out, and it is comprehended as seemingly taking the form of the collection (of the word-meanings). (VP II 143-5, Pillai)

The important feature of this description is that no sphoṭas are described in it to bear the sentence-meaning. The sentence-meaning arises directly from word-meanings, as the word-meanings arise from letter-meanings in the VS' grammar. It is possible that Bhartṛhari might have omitted to mention a sentence-sphoṭa in this description, but it does not seem likely. My conclusion is that Bhartṛhari held what I have called the identity theory of sphoṭa and meaning, that there is textual evidence in this passage to that effect, and that the reading of other texts is less problematic if we ascribe the

identity theory to him.

It remains to explain the attraction of the non-identity theory to Bhartrhari's successors and commentators. Their view seems to arise from a concentration on the facts of language-processing at the word level, without comparable attention paid to such facts at the sentence level. A word-meaning considered in isolation has a name--the concatenation of the English transliteration, in VS. It is common when reading a word (especially a word in isolation) to have the name come to mind when the word is understood, and it is possible to take this name to be expressive of a word-form, so that one might take sounds or inscriptions, forms, and meanings to be involved in the processing of a word. This way of understanding what happens is wrong if the identity theory is correct. According to the identity theory, one reads inscriptions, has a meaning in mind, and a subvocal name of the meaning comes to mind, at the same time as the meaning or just after. The subvocal name is a series of sounds in form, and it expresses the word meaning, but it appears to be a new form which is used in the processing of language. Hence one might be tempted to believe that a word-sphota comes to mind before word meaning does.

This temptation disappears when sentence meanings are considered, for we do not have names for sentence meanings which are so obviously linked to them. What we use to name sentence meanings are sequences of words. But sequences of words are taken to be related first to word-meanings and only derivatively to sentence meanings. Concerning the fact that sentences do not have names in the way that words have names, Bhartrhari writes:

(22) We do not investigate the real nature of that (here, the word-meaning) which (obviously) has a form of its own. It is only when a thing (here, the meaning of the sentence) does not have such a form that we seek for its essential nature. (VP II 416 Pillai)

To realize that sentence-meanings are entertained in the mind without uncompound sentence-names to act as their "forms" is to lose the temptation to suppose that there must be sentence-sphoṭas in addition to sentence-meanings when speech is processed.

The moral is that commentators have concentrated on the sort of sphoṭa which we describe best, word-meanings. When attention is paid to passages such as that quoted in (21), the temptation to think that there must be a form mediating between word-meanings and sentence-meanings disappears. Bhartṛhari saw deeply into the relation between sounds and meanings, so deeply that he realized the identity theory to be true. I believe his successors and many of his commentators to have lost the depth of his vision.

4. Comments on the Sphoṭa Theory

Bhartṛhari's theory of sphoṭa contained several important insights. One, that there is a difference in kind between speech sounds and meanings, has been discussed. Some of the others deserve mention, although they will not be treated in detail here.

Meanings do not have parts. One feature of sphoṭas which Bhartṛhari mentions again and again is that they are noncomposite entities. Some passages in which this point is made are:

(23) Just as there are no parts in letters (similarly) there are no letters in the word. Nor is there any reality in abstracting the word from the sentence. (VP I 73, Pillai)

(24) The sentence which is (really) indivisible becomes capable of division when it is (analytically) conceived and due to this the meaning which is, in fact, indivisible, is presented to the mind as if it consisted of parts.
(VP II 27, Pillai)

Although the entities which express a meaning may be many, it is continually stressed that the meaning expressed is a single entity. This view, in contrast to a view of the Mīmāṃsakas that the meaning is made up of the concatenation of word-meanings, represents a great insight.

Sentences do not have parts. The sense of "sentence" (vākya) in (24) differs from that in Chapter I. There a sentence was taken to be a sequence of words, an entity which is eminently divisible. Here it is taken to be the sentence meaning. This theory constitutes Bhartṛhari's official definition of a sentence. That it is is stated in passages such as (24). The theory (as I interpret it) is that sentences are not <u>really</u> sounds or inscriptions, although we talk as if they were for simplicity's sake. What they are <u>really</u> is what the sounds or inscriptions express--meanings. There are some interesting consequences of this theory. One is that when syntacticians discuss the syntactic behavior of words they are not really talking about entities which are divorced from semantics, since what words name is <u>meanings</u>. In the VS' grammar, the only entities which are nonsemantic are the inscriptions with which the grammar begins. The rest is meaning, although words are used to talk about what the meanings are like.

Another consequence of this theory is that one may have sentences in mind even when one is not capable of using language. This is the

explanation of the apparently puzzling passage:

(25) In this world no comprehension is possible except as accompanied by speech. All knowledge shines as permeated by speech.

This speech exists within and outside all living beings. Consciousness can exist in all creatures only after it is preceded by speech. (VP I 123 and 126, Pillai)

The relation between speech and consciousness in Bhartṛhari's thought deserves a separate volume of its own.

The act of understanding meaning is unknowable. One provocative feature of the sphoṭa theory is the remark quoted in (21) to the effect that the flash of insight which is the act of understanding a sentence meaning is not known by us. We know that it occurs and we have names for the meanings grasped in such acts but we seem to have no insight into the nature of the acts themselves: "Having been formed from the function of one's inner self, its nature is not known to the person." A discussion and investigation of this doctrine (with its relation to remarks of Western philosophers such as Wittgenstein and Putnam that meanings are not mental entities) is a fertile subject for research.

The relative reality of words and sentences. A theme which Bhartṛhari takes up again and again in Book II of the Vākyapadīya is the claim that sentences are real while words are not. Consider this passage:

(26) Those who consider the sentence as an indivisible unit consider (the recognition of) words (in it) as pragmatic and as subsequent to indivisibility (in the order of reality). (VP II 57, Pillai).

This theory is based on the claim that sentence-meanings are the primary units of thought. Since there are, strictly speaking, no words in the sentence, there are no words in thought. The claim

is made in various ways and seems to take on various forms. A good portion of Book II is taken up with arguments for Bhartrhari's theory that words are not real, and by consideration of counterarguments advanced by the Mīmāṃsakas to the effect that words are ultimately real and sentences are not. These disputes, extremely involved and in need of clarification, comprise another subject for future work.

FOOTNOTES

1 Most of the passages quoted here were gleaned from similar discussions in Raja (1963), pp. 140-45 and Brough (1951), pp. 405-411.

2 That Bhartrhari would agree to a grammar similar to VS as one formalizing the concept of letter-_sphotas_ is not clear. Often it seems that he denies that letters have any meaning, but I take it that he does so for the same reason he often denies that words have any meaning--on his theory only sentences have _real_ meaning. The VS grammar is intended to model a Grammarian view that letters have _sphotas_ associated with them. Whether it is related to a text of the _Vakyapadiya_ I am not certain, although there are discussions of the way letters form words which might be illuminated if referred to a grammar like the VS grammar.

C H A P T E R I V

BHARTṚHARI AND THE LIAR PARADOX[1]

In chapters I-III some Indian concepts have been rendered in Western formalism. In this chapter I shall hold the position that another sort of Western formalism, that usually employed to represent positions like Bhartṛhari's, is not needed. The sort of formalism I have in mind is the apparatus of language hierarchies customarily used in formal semantics. I shall argue that to use such apparatus is to rob Bhartṛhari's remarks of their import, and shall present an alternative system within which the views Bhartṛhari held may be expressed as he meant them.

1. Tarski's Proof and Tarski's Solution[2]

The theory of languages and metalanguages constitutes the response of the Western logician Alfred Tarski to a problem raised by a result he discovered. The result (hereafter, "Tarski's Proof") was based on these two assumptions: The truth predicate is bivalent (that is, every sentence is either true or false), and Convention T--for any constant a of a language L which names a sentence q such that p is the translation of q into L, the biconditional of the following form is true in L: a is true if and only if p.

Using these assumptions, Tarski proved that any language which contains the usual logical operators, names of its sentences, and a truth predicate will contain sentences the assignment of truth values to which will lead to contradiction. (For a language with names of

its own sentences, the translation of each sentence will be itself). Such sentences are commonly called Liar sentences. A notorious example is the Liar, "This sentence is false".

Tarski's Solution to this problem, as it is often informally stated, is to require that no language we employ contain its own truth predicate. More precisely, this solution is often stated as the requirement that we employ only those languages which form metalanguage hierarchies. (As we see shortly, these two ways of putting matters are not equivalent.) A full specification of what a language hierarchy is, however, is not given us by Tarski, and in order to make some points clearly later on, such a specification is proposed here.

First of all, some assumptions. It is assumed that the languages under discussion contain sentences made up of quantifiers, logical operators, constants, and predicates, although none of the languages is required to contain expressions from all these categories. To make proofs simpler, the quantifier is assumed to be the existential quantifier and the logical operators are assumed to be "$\vee$" and "$\sim$". It is assumed further that the well-formedness conditions on sentences make an n-place predicate of a language followed by any n constants of that language a sentence. It is assumed that the complex sentences are built up from the quantifier and logical operators in the usual way. It is assumed that the languages under discussion are interpreted. That is, each language has associated with it an ordered pair $\langle D,V\rangle$, where D is a set comprising the domain of interpretation and V is a function assigning to each constant of the language a member of D, and assigning to each n-place predicate of the language a subset of the

n-term Cartesian Product of D. It is assumed that any set of languages which forms a language hierarchy will not be a set in which the assumptions made in Tarski's Proof lead to contradiction. (This assumption justifies some of the requirements to be made on language hierarchies.) The definition of a language hierarchy follows.

(1) A language hierarchy H is any ordered pair $\langle L, M \rangle$ such that L is a set of languages and M is a relation from L to L such that for any L_i and L_j, members of L,

(a) the domain of interpretation of L_i contains some sentence of L_j and L_i contains a truth predicate if and only if ML_iL_j,

(b) if $L_i \neq L_j$ then there is a sequence of languages of L $L_1 \ldots L_n$ such that ML_iL_1 or ML_1L_i, ML_1L_2 or ML_2L_1, ... $ML_{n-1}L_n$ or ML_nL_{n-1}, and ML_nL_j or ML_jL_n,

(c) all sentences in the union of the domains of interpretation of the members of L are sentences of some member of L,

(d) there is no sequence of members of L $L_1 \ldots L_n$ such that ML_1L_2, ML_2L_3, ... $ML_{n-1}L_n$ (hence, trivially, M is irreflexive),

(e) there is no infinite sequence of members of L $L_1 \ldots L_n$ such that, for each L_i in the sequence, ML_iL_{i+1}.

Let us read "ML_iL_j" as "L_i is meta on L_j". It may be surprising that the conditions in (1a) for one language to be meta on another are so weak. One does not encounter languages in Tarksi's work or in the literature in which a language meta on another contains a name of no

sentence of the second. There is a reason for this. Tarski writes in terms of the metalanguage relation, perhaps believing that it was the only relation which satisfied his intuition that no language should contain its own truth predicate. The metalanguage relation is more restrictive than the relation of being meta on. As Tarski characterizes it,

(2) L_i is a metalanguage for L_j = L_i contains a name of every sentence of L_j and L_i contains a truth predicate.

The context in which language hierarchies are most frequently encountered is that in which a semantics for an object language is being given in a metalanguage. Tarski believed that, in order to state a semantics, it is necessary to give a sentence of the form

(3) a is true if and only if p

for each sentence of the objectlanguage. Hence each object language sentence must have a name in the metalanguage. I have adopted this requirement on the semantics to be given later, although I think it is overly restrictive. A semantics, in the sense of a complete specification of the truth conditions for the sentences of an object language, can be given in a language which has no names for the object language sentences. For example, I believe the truth conditions on propositional calculus sentences are stated accurately in this way:

(4) (x)(x is true iff the last column of a truth table for x contains "T" on every line)

Provided the domain of interpretation of the language (4) is a sentence of contains each sentence of the propositional calculus, (4) gives the truth conditions for those sentences correctly. It is not required

for adequacy that the language (4) is found in contain names for those sentences as well.

If the metalanguage relation is substituted for the relation of being meta on in (1a), the resultant structures have the characteristics which are discussed with respect to language hierarchies. However, it seems interesting to point out that the two intuitions spoken of as Tarski's Solution differ in practice. The language hierarchy definition is intended to be a formalization of the intuition that no language contain its own truth predicate, and the resultant structures have the set of metalanguage hierarchies as a subset.

The characterization of M in (1a) is not a syntactic one because not all Liar paradoxes arise from syntactic self-reference. The sentence

(5) $(Ex)(Px\&\sim Tx)$

may be an instance of the Existentially Quantified Liar, if T is the truth predicate and P is a predicate satisfied only by (5). There is no syntactic hint that this is the case; only the interpretation of the language containing (5) determines whether a Liarlike sentence is in question.

Requirement (1b), that M connect all the members of a language hierarchy, is not essential to the results to be discussed here, but without it any set of languages would constitute a language hierarchy, given only that each lacked a truth predicate, and this seems contrary to the connotation of "hierarchy".

Requirement (1c) ensures that there will not be reference to sentences outside a language hierarchy. Otherwise, inference rules

for the truth predicate could lead to paradox in the Liarlike way.

Requirement (1d) rules out Liarlike situations of the following sort:

(6) Ta_1 (sentence of language L_3)

(7) Ta_2 (sentence of language L_4)

(8) $\sim Ta_3$ (sentence of language L_5)

where a_1 names (7), a_2 names (8), and a_3 names (6). The assumptions of Tarski's proof will produce a contradicition if these sentences are members of a language hierarchy.

Requirement (1d) ensures also that no language of a language hierarchy contain its own truth predicate. A consequence of (1d) which Saul Kripke disparages in Kripke (1975) is that syntactic self-reference is prohibited as semantic self-reference is prohibited, through the joint effect of (1a) and (1d). However, given Tarski's requirement that languages not contain their own truth predicates and the assumption that any n-place predicate of a language followed by n constants of that language is a sentence of that language, this result follows. Kripke's interpretation of Tarski's Solution differs from mine on this point, but I cannot specify precisely the way in which we differ, because Kripke doesn't give his version in Kripke (1975), due to limitations of space.

Requirement (1e) ensures that the hierarchies will be grounded. This requirement is not a feature of the system given later, but it seems necessary if Tarski's system is to preserve bivalence.

One could require that each hierarchy contain at least two members, but this does not seem to be essential to the notion of a

hierarchy. A single, truth-predicateless language which discusses cows seems acceptable as a degenerate hierarchy. (Nothing to come seems to depend on this decision.)

Now it may be demonstrated that Tarski's Solution works in this form. Let the standard semantics for each sentence w of a language L_i of a language hierarchy H be as follows (where V_i is the second member of the interpretation of L_i):

(9) if w is of the form $p^n_m a_1 \ldots a_n$, w is true if and only if $\langle V_i(a_1), \ldots V_i(a_n)\rangle \in V_i(p^n_m)$.

(10) if w is of the form $\sim p$, w is true if and only if p is not true.

(11) if w is of the form $p \vee q$, w is true if and only if p is true or q is true.

(12) if w is of the form Ta_n, w is true if and only if $V_i(a_n)$ is true.

(13) if w is of the form $(Ex)(\emptyset x)$, w is true if and only if some member $v \in V_i$ satisfies $\emptyset$.

(14) any sentence p is true if and only if p is not false.

By applying (9)-(14) to the sentences in the lowest members of any language hierarchy and working upwards in the hierarchy, each sentence of each language receives a truth value.

Contradictions do not result from applying the assumptions of Tarski's Proof to any language hierarchy, if there was no contradiction derivable in the languages of the hierarchy through usual rules of deduction. The proof of this statement is as follows. Let us assume a set of deduction rules which apply to the sentences of

each language in a language hierarchy H. Let us assume that the rules are consistent--that is, using the rules with true sentences of any language of H as premises yields no contradiction in L_i. Tarski's Proof uses this additional premise having to do with relations between languages in H:

(15) For any constant a_n and sentence q, where "Ta_n" is a sentence of L_i and $V_i(a_n)$ is q and p is a translation of q into L_i, Ta_n if and only if p.

This premise (Convention T) left to right yields no additional premises for the deduction rules to use, for if Ta_n is a truth of L_i, q is a truth of some other language L_j lower in the hierarchy. By hypothesis, p is the translation of q into L_i. A condition of adequacy on translations is that the translations have the same truth conditions as the original sentence. (Tarski supposes the translations of object language sentences into metalanguages to be themselves. I am not certain that his description is the best one, but the point about translations is demonstrated by it.) The truth of q, then, implies the truth of p.

Convention T right to left also yields nothing new. Suppose p is true in L_i. Then, by the conditions on translations, q is true in L_j. By the conditions on the standard semantics, Ta_n is true in L_i. Hence no new truths are added to the sentences of a language by Convention T, and no contradiction is derivable in a language of H if no contradiction was derivable in that language without the use of Convention T.

Tarski's Solution works. Placing the truth-predicate for each language in a language meta on it and imposing requirements (1a)-(1e)

on the relation M eliminates the possibility of contradictions being derived from the assumptions of Tarski's Proof, given a standard semantics and a consistent set of deduction rules.

This is the comforting feature of Tarski's Solution. It has, however, some uncomfortable features for philosophers of language in general and for Bhartṛhari in particular. Let us consider them.

2. Some Drawbacks to Tarski's Solution

Tarski's Proof convinced Tarski that English (as well as Sanskrit and every natural language with names for sentences, a truth predicate, and a negation operator) is inconsistent. For this reason he gave up the use of natural languages, preferring to use only formal languages in whose consistency he had more confidence. There are some difficulties with this move on his part, however. In stating the fact that English is inconsistent Tarski used English. If his theory is correct, the sentence "English in inconsistent" is a consequence of his reasoning, but the sentence "English is consistent" follows from it as well, as does the negation of each of his statements about his solution! (This is a trivial consequence of the fact that any sentence is implied by a contradiction. Given that Tarski has demonstrated a contradiction in English, he has demonstrated that every sentence is a consequence of the "truths" of English.)

A natural reply is that Tarski was using a metalanguage for English (call it "L") in carrying out his proof, so that the inconsistency in English would not infect his results. It is open to us, then, to wonder where L comes from, and how Tarski came to speak it. L cannot

have been created by thoughts in English (or Polish; assume one natural language to be in question) of the sort "Let there be a language L such that in L sentences of the form P&Q are true if and only if P is true and Q is true." If this were the case, English would be meta on L (by (1a)) and Tarski could not use L as a metalan guage for English without violating the irreflexivity conditions on the relation M. How is L to be learned by an English speaker so that it may be a metalanguage for English? The answer is not clear.

Suppose, however, there to be a language hierarchy containing some constrained version of English which has a truth predicate--call it "T-English". There are other difficulties. For example, the sentence

(16) This sentence is in T-English.

cannot be a sentence of T-English. (To allow it would be to make T-English meta on itself and violate condition (1d) on relation M).

Another drawback is that the T-English sentence

(17) Any sentence which is a conjunction of two true conjuncts is true, and any sentence which is a conjunction of two false conjuncts is false.

applies to sentences of languages T-English is meta on, but (17) cannot apply to (17) itself, for the sentences of T-English cannot be in the domain of quantification of T-English. Thus Tarski's Solution rules out discussion of the nature of all languages if the nature discussed is semantic. (There is some support here for the thesis sometimes held by linguists that the semantic and syntactic components of language may be described independently. If Tarski's

Solution is correct, the syntax of a language may be discussed in that language, but the semantics must be discussed in another language.)

Another drawback is illustrated by this sentence:

(18) One is able to state the semantics for T-English in the formal languages one learns in logic classes.

Given Tarski's Solution this sentence is false, for the reasons just discussed. What Tarskian semanticists view themselves to be doing with the formal languages learned (in English) in logic classes is carrying out a program based on a more restricted view of English:

(19) What we call English is actually a series of metalanguages which appear to be the same language. We are able to describe formal languages in some of these metalanguages and use the languages described to do semantics for others of these metalanguages lower in the hierarchy.

Several difficulties afflict this view of things. First, on this view it is often the case that users of English do not know what language they are speaking. For example, the statement

(20) Some sentence in the New York Times today is false.

is required to be in a language at least one level higher in the hierarchy than the level of the highest-level language which has a sentence in today's New York Times. A person could (and often does) assert (20) without knowing the level of the sentences discussed.

Another difficulty would seem to lie in acquiring the languages of the hierarchy. There must be infinitely many languages making up the set of languages which are English as characterized in (19), and the miracle of learning one language is multiplied beyond comprehension if (19) is correct. Furthermore, (19) does not seem to fit the facts. The sentence

(21) The next numbered sentence in this chapter is true.

is a sentence of English. Suppose this to be the next numbered sentence:

(22) The previous numbered sentence in this chapter is false.

To suppose these sentences part of a language hierarchy is to violate the requirements on M. If English is a series of languages of a hierarchy, then (21) and (22) are not sentences of English. The most telling difficulty, however, is that the language hierarchy solution in general as well as this instance of it leads to the partitioning of our utterances into langauges on the basis of the way we use the truth predicate. Such partitioning has no basis, as far as I can determine, in the way we think. When we go through a process of converting the thoughts expressed by (16), (17), and (18) into sentences we follow the same set of rules. It would be possible to follow a different set of rules and arrive at French or Sanskrit sentences expressing our thoughts, and if we did so we would say that we were using a different language, or that the results were in a different language from that in which (16), (17), and (18) are stated. But (16), (17), and (18) contain words drawn from the same lexicon, they obey the same syntactic rules, and our obvious inclination is to say that they are in the same language--English. Nothing in the procedure of getting from meanings to words gives us a basis for putting them in different languages. The Solution advocated by Tarski requires that we jettison a good many of our beliefs about thought and language, with no basis for the jettisoning other than the results in Tarksi's Proof.

Given all this, one might wonder why the language hierarchy apparatus is used in Western semantics. The answer has two parts. First, no better way to avoid the paradoxes discussed in Tarski's Proof has been known until recently. Second, some attempts have been made to weaken Tarski's assumptions and allow for languages to contain their own truth predicates without contradiction. In the next section some of these attempts are discussed.

There are several features of Bhartṛhari's theory of language which are metalinguistic, in the sense that a Tarskian semanticist would use language hierarchies to formalize them. In the last chapter Bhartṛhari's theory of sphoṭas was discussed. On a Tarskian view, to propose a theory about the meaning of a sentence is to discuss the conditions under which the sentence is true. Bhartṛhari's theory was one which had to do with all sentences of Sanskrit and their meanings. (Perhaps all sentences of every language as well, but no text bears on this point.) It follows that a Tarskian rendering of his remarks would place them in a language meta on Sanskrit. (If the view in (19) is adopted, then the solution would be to place them in a language meta on the languages of the Sanskrit hierarchy). This solution will not do, however, for Bhartṛhari's remarks were in Sanskrit and the point he intended to convey applied to the sentences in which the theory was expressed as well as the other sentences of Sanskrit. A translation of his theory which fails to make it applicable to the theory itself is not an adequate translation.

There are passages in which Bhartṛhari discusses other semantic systems. These passages are considered in the next chapter at greater

length. Here we examine one example of the metalinguistic sort.

One school of Indian philosophers, the Nyāya-Vaiśeṣikas, gave a series of categories into which all existing things were to fall. One such category was the category of universals. The Nyāya-Vaiśeṣikas claimed that universals are what are in common in similar objects. They also argued that universals themselves do not have universals, since to allow this would be to allow an infinite regress of universals, and it was a tenet of the Nyāya-Vaiśeṣika system that there is no infinite regress of existents.[3]

Bhartṛhari's view of universals was that they occur in individuals and that they cause our cognitions of similar things to be similar. A part of the theory which was held to be central was that whenever we cognize things as similar, we do it because there is a universal in those things which causes the cognitions. Bhartṛhari noted that we cognize universals as similar (this is the basis for grouping them in the same category) and concluded that there must be a universal of universals, and a universal of the universals of universals, and so on. This conclusion contradicted the Nyāya-Vaiśeṣika claim that there are no universals of universals. One tenet of the Grammarian school in Bhartṛhari's time was that the Grammarian philosophy should be common to all the known philosophical schools. This contradiction, then, required discussion.

First of all, Bhartṛhari attempted to resolve it by distinguishing two senses of "universal"--word-universals and real universals. I do not understand how this solution was intended to work. The discussion of it is found on pp. 10-14 of Iyer's translation of Book III of the

Vākyapadīya. An example follows:

(23) The universal of words, residing in words and differing from the words themselves, includes also the universal existing in the word śabdajāti ('word-universal')
(VP III 1.10, Iyer. Parenthetical remarks mine.)

Helārāja's commentary on this passage, as summarized by Iyer, is as follows:

(24) The universal which exists in words and which is different from the words themselves is so comprehensive that it includes the universal which exists in the word śabdajāti itself. When we say: śabdajāti, it does not denote a universal which is over and above such universals as gośabdatva ('cow-word-ness'), aśvaśabdatva ('horse-word-ness'), etc. Because, one does not believe in a universal over a universal. So the universal which exists in the word śabdajāti itself does the work of such a universal. The universal which exists in the word śabdajāti is in the same category as the universal which exists in the word gauḥ ('cow') or aśvaḥ (horse).
(VP III 1.10, Helārāja's commentary. Iyer.)

What this attempt to achieve compatibility with the Nyāya-Vaiśeṣikas amounts to, I am not certain. What is certain is that Bhartṛhari is not ultimately in sympathy with it. Having struggled as we have seen to make the positions compatible, Bhartṛhari proposes another account of the incompatibility:

(25) ...the alternative is to say that words denote the universals of the object, pure and simple. Even then, one will have to maintain that all words do so....Here one has to meet the Vaiśeṣika objection that if the word jāti [universal] also stands for a universal, it must be a universal existing in other universals. And that is not possible. If there is universal [sic] in universals, where would one stop? But Grammarians have a different point of view. Their chief concern is to find out the nature of meanings conveyed by words. What they find is that in all universals as conveyed by words, there is a common point or characteristic which can be looked upon as another universal and can be called by the name jāti.... Grammarians go by what the words convey. They are not really concerned with things as they really are, but with things as conveyed by words. If a quality is conveyed by words as a common characteristic, it becomes a universal for them. In such matters, grammarians are more anxious to follow worldly

> usage than the views and conventions of other systems of thought. To them, artha ('meaning') means what words convey and all words convey the universal, because that is a matter of fact. In reality, the universal which a word conveys may or may not exist. But the word in question, through its function called abhidhā [sense] does convey it. So that is the meaning of the word.
>
> (Summary of Helārāja's commentary on VP III 1.11, Iyer.)

These points have to do with the correct assignment of meaning to the word jāti (universal). Again, it seems that Bhartṛhari's remarks must be formulated in a metalanguage for Sanskrit. The Nyāya-Vaiśeṣika theory includes a list of categories which include the denotations of all words. According to the theory the sentence "The pot is red" means something like this: "Redness inheres in the pot-substance". There are similar translations for other sentences. Therefore the Nyāya-Vaiśeṣika system is a semantics for Sanskrit, given the assumption that to specify meanings is to specify truth-conditions. Bhartṛhari, in discussing the truth of the Nyāya-Vaiśeṣika theory, is discussing the meaning of a term which must be in a metalanguage for Sanskrit. His position, that if we take jāti to refer to what words present to our minds then the Nyaya-Vaisesika theory is false and if we take jati to refer to something in reality then perhaps it is true, must be in a language meta on the metalanguage for Sanskrit in a Tarskian interpretation. This treatment again does not adequately formalize the theory, for the Nyāya-Vaiśeṣikas' remarks are intended to apply to the sentences in which Bhartṛhari states his theory of universals. As we have seen, there is no way for a Tarskian account to allow for sentences which refer to each other in a language hierarchy.

There is another reason Bhartṛhari's remarks would not be rendered in a way which he would have approved of if they were formulated in a

language other than Sanskrit. Bhartṛhari held that the only language worth speaking in was Sanskrit. Although other languages have been devised and meanings are conveyed in them, their use does not lead to spiritual advancement. If there is a way to represent Bhartṛhari's views, then, so that they turn out to be couched in Sanskrit and so that the paradoxes of Tarksi's Proof are evaded, it would be preferable to that of a Tarskian semanticist. In the remainder of this chapter such a way is proposed and discussed.

3. How to Create the Meta-effect Without Meta-Apparatus

Some Western logicians have weakened the assumptions of Tarksi's Proof in order to devise langauges which contain their own truth predicates without contradiction. One such system has recently been proposed by Saul Kripke in Kripe (1975).[4] There Kripke constructs a consistent semantics for a language which contains its own truth predicate by denying Tarski's assumption that every sentence is either true or false. As we have seen in the discussion of yogyatā, this assumption has been denied by some logicians on other grounds, and Kripke's solution has some intuitive appeal. There is difficulty with it, however. The system is described in a metalanguage containing predicates which the semantics does not treat (the predicate "undetermined in truth value", for instance). Whatever its merits, it is unsatisfactory that the system is couched in metalinguistic terms which the semantics cannot treat. Further, given that the metalanguage apparatus is used by Kripke, the problem of identifying the languages

which he uses and their relation to ordinary English is as difficult for his system as it is for Tarski's.

The difficulty with the Tarski system and Kripke's as well is that they are based on Convention T. In Kripke (1975) Kripke writes,

> (26) We may say that we are entitled to assert (or deny) of any sentence that it is true precisely under the circumstances when we can assert (or deny) the sentence itself. (p. 701)

Convention T is actually two theories about truth. One is that "a is true" entails p, where p is the translation of the sentence a names; this part of the theory is the disappearance theory of truth. The other part is that, for any sentence p, where p is the translation of the sentence a names, p entails "a is true". This theory is the appearance theory of truth. If one accepts both these theories it follows that no statement of the standard semantics of a language containing the logical operators already described is true if formulated in the language it is a semantics of.

In order to prove this statement, suppose it false. Then the semantic statement s for a language L is a sentence of L. The semantics includes the following clause: All sentences of L are true if and only if ___ (some formula or other fills in the blank). According to the standard semantics, the truth value of a quantified sentence depends on the satisfaction of the formula by each member of the domain. One such member is s. The truth value of s, then, depends on the truth value of

(27) s': s is true if and only if ___.

But s' is true if and only if the sentence "s is true" agrees in truth value with the sentence on the right of the equivalence sign. We must,

therefore, know the truth value of "s is true" in order to know the truth value of s'. But that can't be obtained without determining the truth value of s. We are involved in an infinite regress and our assumption is false.

Given that Bhartṛhari takes his theory to be discussing semantic theories of Sanskrit and that he takes it to be given in Sanskrit, no theory which places his remarks in another language will seem adequate to formalize his theory.

It seems, then, that a different approach is needed here. Such an approach will be given in the remainder of this chapter. The basis for the approach is that the contradictions derived from Tarski's assumptions constitute a reductio ad absurdum of those assumptions, rather than a proof of the intrinsic inconsistency of natural language. If Convention T entails the inability of natural language speakers to do semantics in their natural languages, then Convention T is suspect. The theory to be given constitutes a formalization of a different view of the truth predicate, one which allows us to say the things about language which we do say, in the languages we say them in.

The semantics is given with respect to a language called "SR" (for "self-referential") The language and the semantics have these features:

Sentences of SR have names in SR,

SR has a truth predicate,

SR has the predicate "Undetermined",

The semantics for SR assigns a unique truth value to every sentence of SR,

Any language hierarchy may be translated into SR so that each sentence preserves its truth value and all inferences in the hierarchy based on Convention T are preserved,

One may correctly assert the SR semantics,

The SR semantics is formulated in SR,

Convention T is not true of the SR semantics, and

The theory of truth upon which the semantics is based is that a sentence is true if the interpretation of SR guarantees its truth, false if the interpretation guarantees its falsehood, and undetermined otherwise.

This view of truth may be characterized as a groundedness theory of truth, as opposed to a Tarskian theory of truth. According to a groundedness theory, to be true a sentence must be grounded in atomic sentences which guarantee its truth value on the interpretation. Kripke's theory is a groundedness theory as well as a Tarskian theory because it does not contain the predicate undetermined. Given a language without this predicate, the two sorts of semantic systems behave alike. With the predicate their behavior diverges, as we shall see.

There are four features of the system which are not generally used. First, the evaluation process employs trees constructed of nodes. Each node contains five members: a sentence of SR, a Semantic Domain, the index T, F, or U, a valuation function, and an integer to distinguish occurrences of nodes with identical first four members. A node on a tree with sentence w, Semantic Domain SD, truth value TV, and function V as first four members directly dominates those nodes with sentences, Semantic Domains, truth values, and functions required

to guarantee TV for w over SD under the interpretation function V. Trees are employed rather than the more common clause-by-clause evaluation procedure because, for cases in which a sentence does not have a truth value guaranteed by the interpretation, certain highest nodes become acceptable which are not acceptable on the interpretation. The tree notation is necessary in order to determine the highest such nodes.

The second respect in which this system differs from the more commonly used semantic systems is that the semantic predicates T, F, and U are two-place predicates rather than one-place predicates. This is a formalization of Tarski's insight that employing the predicate true requires separating sentences into groups--those which may be referred to semantically and those which may not. The sentence "Tab" is to be read as "a is true over Semantic Domain b". This means that in the evaluation of a, a's truth value may not be established by reference to sentences which semantically discuss sentences not in b. Restriction of the domain of the semantic predicates by means of an argument place rather than by partitioning sentences into language results in the creation of the metalanguage effect without the employment of language hierarchies. (Note: in this paper I interpret the ordinary language truth predicate "x is true" as "TxW", where W is the set of all sentences in the domain of interpretation.)

A third difference is that the evaluation process assigns a Semantic Domain to each sentence at each stage of the evaluation. Formally, this is accomplished by including the Semantic Domain as the second member of each node of a tree. The Semantic Domain of any

sentence w of a node n is the same as that of the sentences in the nodes which n directly dominates unless w is an atomic semantic sentence--that is, w is of the form Tab, Fab, or Uab. Then the Semantic Domain of the nodes n directly dominates is b less the set of sentences containing semantic discussion of the sentence a names, sentences which have those sentences as constituents, sentences semantically discussing any sentences in this set, and so on. Formally this new Semantic Domain is represented as b-a*. The effect of removing a* from b is that all sentences which would, in the Tarskian view, be required to be in languages meta on the language containing Tab, Fab, or Uab are made inaccessible to semantic predication as a is evaluated. This is the SR analogue to moving to a lower language in the Tarski hierarchy, but again the effect is achieved as a part of the evaluation process rather than by a prior partitioning of sentences into languages.

The fourth respect in which the SR semantics differs is that the nodes of trees receive acceptability labelling. A tree which establishes the truth value of a sentence of SR over a given Semantic Domain must have the features listed above, and it must be acceptable. That is, it must assign to non-semantic atomics truth values based on the interpretation of SR so that the topmost node assigns a truth value compatible with the interpretation. Acceptability is determined by labelling terminal nodes according to the interpretation, and then labelling upward if possible. It is necessary to use acceptability labelling because some subtrees used in the semantics are trial trees. For example, a node n of the form $\langle$Uab,SD,T,V,i$\rangle$ directly dominates two nodes of the form $\langle$V(a),b-a*,T,V,j$\rangle$ and $\langle$V(a),b-a*,F,V,k$\rangle$.

If one of these nodes is labelled acceptable, then n is labelled not acceptable, for the interpretation establishes a truth value for a. If both nodes directly dominated by n receive no label then n is labelled acceptable, for the interpretation has failed to determine a truth value for a.

Here are two examples of the way the semantics works.

Let s be "Rāma is a man or Rāma is not a man." Let s be symbolized in SR as "nara Rāma∨~nara Rāma". Then an acceptable semantic tree for s is the following (where nodes are labelled n_i for reference, W is the set of all SR sentences, and the relation of direct dominance is represented by a line downwards from the dominating to the dominated node):[5]

(28)

n_1 ⟨TsW,W,T,V,1⟩

n_2 ⟨nara Rāma∨~nara Rāma,W-s*,T,V,2⟩

n_3 ⟨nara Rāma,W-s*,T,V,2⟩ n_4 ⟨~nara Rāma,W-s*,T,V,5⟩

n_5 ⟨nara Rāma,W-s*,T,V,4⟩

Nodes n_3 and n_5 of (28) contain sentences which are atomic. If they are interpreted in the intended way, n_3 will be an acceptable node (since it is true that Rāma is a man). This establishes that n_2 is acceptable, and this establishes that n_1 is acceptable. The fact that the semantic domain has changed makes no difference, since s involves no semantic predication. The labelling would be the same for "s is in SR or s is not in SR".

Let s name the Liar sentence "~TsW" and let U be the predicate undetermined. An acceptable semantic tree for s is the following:

(29)

n_1 ⟨UsW,W,T,V,3⟩

n_2 ⟨~TsW,W-s*,T,V,2⟩ ⟨~TsW,W-s*,F,V,16⟩ n_3

n_4 ⟨TsW,W-s*,F,V,2⟩ ⟨TsW,W-s*,T,V,4⟩ n_5

In n_1 it is asserted that s is undetermined. The nodes n_2 and n_3 are attempts to establish a truth-value for s--the trial trees for s. If they are not acceptable, n_1 will be labelled acceptable. The Semantic Domain in n_2 and n_3 has shrunk in moving down from n_1, and in this case it makes a difference, for n_4 and n_5 are terminal nodes. The nodes they would naturally dominate are of the form ⟨~TsW,W-s*,F,V,i⟩ and ⟨~TsW,W-s*,T,V,j⟩, but the first members of these nodes are not in the Semantic Domain of n_2 and n_3. For this reason the tree terminates at nodes n_4 and n_5, which receive no labels. Hence nodes n_2 and n_3 receive no acceptability labels and n_1 is acceptable. In SR the Liar sentence is undetermined in truth value.

4. The Syntax and Semantics of SR

This section is rather formal. The reader who wishes to skip it and take the preceding remarks on faith may wish to go directly to section 5.

Syntax of SR.

Symbols:[6]

1 predicates of the form P^n_m and $P^n_{m,i}$

2 constants of the form a_n and $a_{n,i}$, where every sentence of SR is named by some constant (perhaps through Godel numbering)

3 variables x_1, x_2, ...

4 logical symbols $\sim$, $\vee$, (E), $\neg$ [7]

5 T, F, and U (atomic semantic predicates)

Well-formed Formulas of SR ("wffs"): The wffs of SR are the members of the smallest set S such that

1 for any predicate P^n and constants a_1, a_2, ... a_n, $P^n a_1 a_2 \ldots a_n$ is a member of S (Let P^n stand for either form of the predicate and the a_i stand for either form of the constants.)

2 if p and q are members of S, $\sim p$, $\neg p$, and $(p \vee q)$ are members of S

3 if p is a member of S, $(Ex_n)(\emptyset x_n)$ is a member of S, where $\emptyset x_n$ is the result of replacing at least one constant of p with x_n and where x_n did not occur in $\emptyset$.

4 Tab, Fab, and Uab are members of S, where a and b are any constants of SR.

It is assumed that SR is interpreted. That is, there is an ordered pair $\langle D,V \rangle$ such that, for every constant a of SR, $V(a) \in D$ and for every predicate P^n of SR, $V(P^n) \in \underbrace{DxD\ldots xD}_{n \text{ times}}$, and such that every wff of SR is in D. Then the following definitions are used in the semantics of SR:

A node is an ordered 5-tuple whose members are: a wff of SR, a set of wffs of SR, "T", "F", or "U", a valuation function, and an integer.

A tree is an ordered pair $\langle N, DD \rangle$, where N is a set of nodes and DD is a relation from N to N, such that

(i) N is not empty,

(ii) for every n and m, distinct members of N, there is a sequence of members of N $n, n_1 \ldots n_i, m$ such that $DDnn_1$ or DDn_1n, DDn_1n_2 or DDn_2, ... DDn_im or $DDmn_i$,

(iii) there is no sequence of members of N $n_1 \ldots n_i$ such that DDn_1n_2, $\ldots DDn_{i-1}n_i$ and DDn_in_1 (hence, trivially, DD is irreflexive),

(iv) there is a node $n \in N$ such that for every node $m \in N$ distinct from n, there is a sequence of nodes $n, n_1, \ldots m$ such that every member of the sequence directly dominates the next.

(Trees are commonly represented as in the preceding section, where the relation of direct dominance is shown by placing line segments between nodes such that, if DDn_in_j, n_j appears below n_i on the tree and there is a line segment from n_i to n_j. This notation makes the relationships of direct dominance easier to grasp at the expense of formal precision. Hence the more abstract notation is employed in the definitions given here and the display notation is employed in examples.)

If DDn_in_j then n_i directly dominates n_j.

If there is a sequence of members of N $n_1 \ldots n_j$ such that each member directly dominates the next and such that every node which directly dominates another node of N has a successor in the sequence, then $n_1 \ldots n_j$ is a dominance path from n_1, and n_1 dominates each member of the sequence.

That node of a tree which satisfies condition (iv) above is the topmost node of the tree.

Any node of a tree which dominates no other node is a terminal node.

A subtree of a tree t is a tree t' such that (i) the first member of t' is a subset of the first member of t, and (ii) the set of dominance paths from the topmost node of t' is identical to the set of dominance paths from that node in t.

If a is a constant naming a wff of SR then a* is the smallest set S of wffs of SR such that

(i) for all b such that V(b)=V(a) and all constants c, Tbc, Fbc, and Ubc are in S,

(ii) if V(b)$\in$S then for all constants c, Tbc, Fbc, and Ubc are in S,

(iii) if $\emptyset$ contains a member of S as an atomic constituent, then $\emptyset$ is in S, and

(iv) if V(b)$\in$S and V(b)$\notin$V(c) then for all constants d, Tdc, Fdc, and Udc are in S.

Let "SD" range over sets of wffs of SR and let "V" range over valuation functions and let "i", "j", and "k" range over integers. Then a semantic tree for SR under an interpretation $\langle D,V\rangle$ is a tree t such that the nodes of t and the relation DD of t satisfy these conditions:

SR1 Every node of the form $\langle P^n_m a_1 \ldots a_n, SD, T, V, i\rangle$ or $\langle P^n_m a_1 \ldots a_n, SD, F, V, i\rangle$ or $\langle P^n_m a_1 \ldots a_n, SD, U, V, i\rangle$ dominates no other nodes.

SR2 A node of the form on the left directly dominates a node or nodes of the form shown on the right:

$\langle \sim p, SD, T, V, i\rangle$	$\langle p, SD, F, V, j\rangle$
$\langle \sim p, SD, F, V, i\rangle$	$\langle p, SD, T, V, j\rangle$

$\langle \sim p,SD,U,V,i\rangle$ $\langle p,SD,U,V,j\rangle$

$\langle \neg p,SD,T,V,i\rangle$ $\langle p,SD,F,V,j\rangle$ and $\langle p,SD,U,V,k\rangle$

$\langle \neg p,SD,F,V,i\rangle$ $\langle p,SD,T,V,j\rangle$

Any node of the form $\langle \neg p,SD,U,V,i\rangle$ is a terminal node.

SR3 A node of the form on the left directly dominates nodes of the form on the right:

$\langle p \vee q,SD,T,V,i\rangle$ $\langle p,SD,T,V,j\rangle$ $\langle q,SD,T,V,k\rangle$

$\langle p \vee q,SD,F,V,i\rangle$ $\langle p,SD,F,V,j\rangle$ $\langle q,SD,F,V,k\rangle$

$\langle p \vee q,SD,U,V,i\rangle$ $\langle p,SD,T,V,i\rangle\langle p,SD,F,V,j$ $\langle p,SD,U,V,k\rangle$

$\langle q,SD,T,V,i\rangle\langle q,SD,F,V,m$ $\langle p,SD,U,V,n\rangle$

SR4 If n is of the form $\langle (Ex)\emptyset x,SD,T,V,i\rangle$ then n directly dominates the largest set of nodes of the form $\emptyset b_k$ such that

(i) the subscript on b is the same for each node of the set,

(ii) b_k occurs in no wff of a node which dominates n,

(iii) k is the lowest subscript consistent with (ii),

(iv) there are no two nodes directly dominated by n differing only in the fifth member,

(v) each V' is like V except that V' assigns a member of D to b_k,

(vi) for any V', if $V'(b_k)$ is a wff of SR and $\emptyset b_k$ contains an atomic constituent of the form Tb_kc, Fb_kc, or Ub_kc, then there is a wff w SD of the form Tac such that $V'(a)=V'(b_k)$, and

(vii) for any V', if $V'(b_k)$ is a set of wffs of SR and $\emptyset b_k$ contains an atomic constituent of the form Tab_k, Fab_k, or Uab_k, then for every such constituent, $V'(b_k)\varepsilon SD$.

If n is of the form $\langle(Ex)\emptyset x,SD,F,V,i\rangle$ then n directly dominates the largest set of nodes of the form $\langle\emptyset b_k,SD,F,V',j\rangle$, where the members of the set satisfy conditions (i)-(vii) above.

If n is of the form $\langle(Ex)\emptyset x,SD,U,V,i\rangle$ then n directly dominates the largest set of nodes of the form $\langle\emptyset b_k,SD,T,V',j\rangle$ or $\langle\emptyset b_k,SD,F,V',m\rangle$ or $\langle\emptyset b_k,SD,U,V',n\rangle$, where the members of the set satisfy conditions (i)-(vii) above.

(Several sets may satisfy these conditions; the largest set is to be the one with the lowest integers as last members of its members.)

SR5 Where V(a) is not a wff of SR or where V(a) is not a member of SD or where $V(b)\not\subset SD$, nodes of the form $\langle Tab,SD,T,V,i\rangle$, $\langle Tab,SD,F,V,i\rangle$, $\langle Tab,SD,U,V,i\rangle$, $\langle Fab,SD,T,V,i\rangle$, $\langle Fab,SD,F,V,i\rangle$, $\langle Fab,SD,U,V,i\rangle$, $\langle Uab,SD,T,V,i\rangle$, $\langle Uab,SD,F,V,i\rangle$, and $\langle Uab,SD,U,V,i\rangle$ dominate no other nodes.

Where V(a) is a member of SD and $V(b)\subset SD$, a node of the form on the left directly dominates a node or nodes of the form on the right:

$\langle Tab,SD,T,V,i\rangle$	$\langle V(a),b-a^*,T,V,j\rangle$
$\langle Tab,SD,F,V,i\rangle$	$\langle V(a),b-a^*,F,V,j\rangle$
$\langle Tab,SD,U,V,i\rangle$	$\langle V(a),b-a^*,T,V,j\rangle$, $\langle V(a),b-a^*,F,V,k\rangle$
$\langle Fab,SD,T,V,i\rangle$	$\langle V(a),b-a^*,F,V,j\rangle$
$\langle Fab,SD,F,V,i\rangle$	$\langle V(a),b-a^*,T,V,j\rangle$
$\langle Fab,SD,U,V,i\rangle$	$\langle V(a),b-a^*,T,V,j\rangle$, $\langle V(a),b-a^*,F,V,k\rangle$
$\langle Uab,SD,T,V,i\rangle$	$\langle V(a),b-a^*,T,V,j\rangle$ $\langle V(a),b-a^*,F,V,k\rangle$
$\langle Uab,SD,F,V,i\rangle$	$\langle V(a),b-a^*,T,V,j\rangle$ $\langle V(a),b-a^*,F,V,k\rangle$

$\langle Uab,SD,U,V,i\rangle$ dominates no other node.

SR6 No node relationships except those specified by SR1-SR5 are allowed in t.

A tree t is an acceptable semantic tree under an interpretation $\langle D,V\rangle$ if and only if t is a semantic tree under $\langle D,V\rangle$ and the topmost node of t is labelled a at some finite stage of Procedure 1 or Procedure 2 below:

Procedure 1. The topmost node n of t may receive a label in this way:

A1 If n is of the form $\langle P^n_m a_1 \ldots a_n, SD, T, V, i\rangle$ label n a if $\langle V(a_1)\ldots V(a_n)\rangle \in V(P^n_m)$ and label n na if $\langle V(a_1)\ldots V(a_n)\rangle \notin V(P^n_m)$.

If n is of the form $\langle P^n_m a_1 \ldots a_n, SD, F, V, i\rangle$ then label n a if $\langle V(a_1)\ldots V(a_n)\rangle \notin V(P^n_m)$ and label n na if $\langle V(a_1)\ldots V(a_n)\rangle \in V(P^n_m)$.

If n is of the form $\langle P^n_m a_1 \ldots a_n, SD, U, V, i\rangle$ then label n na.

If n is of the form $\langle Tab, SD, F, V, i\rangle$ or $\langle Fab, SD, T, V, i\rangle$ or $\langle Uab, SD, F, V, i\rangle$ and V(a) is not a wff of SR or V(b) is not a set of wffs of SR then label n na.

If n has as first member a wff of the form Tab, Fab, or Uab and V(a) is a wff of SR which is not a member of the Semantic Domain of n or V(b) is not a subset of the Semantic Domain of n, then n receives no label.

If n is of the form $\langle \neg p, SD, U, V, i\rangle$ or n is of the form $\langle Uab, SD, U, V, i\rangle$ then label n na.

Next step: if n has received no label through application of A1, consider each terminal node on a dominance path of length 2 from n. Label each such node according to A1. Then use A2-A6 to label the nodes which dominate the nodes just labelled. For each such dominating

node m,

A2 If m is of the form $\langle \neg p, SD, T, V, i \rangle$ label m a if some node m directly dominates is labelled a. Label m na if both nodes m directly dominates are labelled na.

If m is of the form $\langle \neg p, SD, F, V, i \rangle$ label m a if the node m directly dominates is labelled a. Label m na if the node m directly dominates is labelled na.

If m is of the form $\langle \sim p, SD, T, V, i \rangle$ or $\langle \sim p, SD, F, V, i \rangle$ or $\langle \sim p, SD, U, V, i \rangle$ then label m a if the node m directly dominates is labelled a. Label m na if the node m directly dominates is labelled na.

A3 If m is of the form $\langle (p \vee q), SD, T, V, i \rangle$ and m directly dominates a node labelled a, label m a. If both nodes directly dominated by m are labelled na, label m na.

If m is of the form $\langle (p \vee q), SD, F, V, i \rangle$ and both nodes directly dominated by m are labelled a, label m a. If either node directly dominated by m is labelled na, label m na.

If m is of the form $\langle (p \vee q), SD, U, V, i \rangle$ and both nodes directly dominated by m with third member U are labelled a, or if one such node with third member U and one such node with third member F are labelled a, label m a. If one such node with third member T is labelled a, or if both such nodes with third member F are labelled a, or if both such nodes with third member U are labelled na, label m na.

A4 If m is of the form $\langle (\exists x) \emptyset x, SD, T, V, i \rangle$ then label m a if some node directly dominated by m is labelled a. If all nodes directly dominated by m are labelled na, label m na.

If m is of the form $\langle (\exists x) \emptyset x, SD, F, V, i \rangle$ and all nodes directly

dominated by m are labelled a, label m a. If some node directly dominated by m is labelled na, label m na.

If m is of the form $\langle(Ex)\emptyset x,SD,U,V,i\rangle$ and one node of each pair of nodes directly dominated by m of the form $\langle\emptyset b_k,SD,U,V',j\rangle$ and $\langle\emptyset b_k,SD,F,V',r\rangle$ is labelled a and there is at least one such node of the form $\langle\emptyset b_k,SD,U,V',s\rangle$ labelled a, then label m a. If some such node of the form $\langle\emptyset b_k,SD,T,V',p\rangle$ is labelled a, or all such nodes of the form $\langle\emptyset b_k,SD,F,V',v\rangle$ are labelled a (where $_v$ need not be the same for each node), or some pair of such nodes of the form $\langle\emptyset b_k,SD,F,V',c\rangle$ and $\langle\emptyset b_k,SD,U,V',d\rangle$ are labelled na, label m na.

A5 If m is of the form $\langle Tab,SD,T,V,i\rangle$ or $\langle Tab,SD,F,V,i\rangle$ or $\langle Fab,SD,T,V,i\rangle$ or $\langle Fab,SD,F,V,i\rangle$, m is not terminal, and the node directly dominated by m is labelled a, then label m a. If these conditions hold and the node directly dominated by m is labelled na, label m na.

If m is of the form $\langle Tab,SD,U,V,i\rangle$ or $\langle Fab,SD,U,V,i\rangle$ or $\langle Uab,SD,T,V,i\rangle$, m is not terminal, and both nodes directly dominated by m are labelled na, then label m a. If these conditions hold and either node directly dominated by m is labelled a, label m na.

If m is of the form $\langle Uab,SD,F,V,i\rangle$, n is not terminal, and some node directly dominated by m is labelled a, label m a. If these conditions hold and both nodes directly dominated by m are labelled na, label m na.

A6 Don't label nodes in any other way.

Next steps: If A2-A6 fail to produce a label for the topmost node of t, label all terminal nodes on dominance paths of length one greater than those just labelled in the previous step. Then label upwards with A2-A5. If no label is produced for the topmost node of t, repreat the process with nodes on dominance paths one greater in length than the paths in the step just completed. Keep doing this. (It may be that this procedure will involve an infinite number of steps.)

Procedure 1 could well fail to produce a label for the topmost node of t. In such a case there is another procedure to apply:

Procedure 2. If Procedure 1 failed to label the topmost node of t by labelling nodes at some finite length d from the topmost node of t and labelling upwards, go through Procedure 1 again with the following change:

Any node of the form ⟨Uab,SD,T,V,i⟩ or ⟨Tab,SD,U,V,i⟩ or ⟨Fab,SD,U,V,i⟩ which failed to receive a label at some finite stage in Procedure 1 is labelled a and the nodes it dominates are not labelled in any subsequent step.

Any node of the form ⟨Tab,SD,T,V,i⟩ or ⟨Tab,SD,F,V,i⟩ or ⟨Fab,SD,T,V,i⟩ or ⟨Fab,SD,F,V,i⟩ or ⟨Uab,SD,F,V,i⟩ which failed to receive a label at some finite stage in Procedure 1 is labelled na and the nodes it dominates are not labelled in any successive step.

Before specifying the semantics for SR by means of these definitions, it is well to relate them to the intuitions discussed earlier. A semantic tree is a display of the wffs and assignments to them which may be required in order to establish a semantic assignment to the wff in the topmost node. Not all nodes of every

semantic tree are necessary to establish a semantic assignment to the topmost node, and not every interpretation agrees with the semantic assignments to wffs in the terminal nodes of a semantic tree. To determine whether a semantic tree is acceptable on a given interpretation, two labelling processes are specified which check the semantic assignments to terminal nodes against the interpretation. In Procedure 1 wffs are checked for groundedness. If they are grounded they are labelled and a label a shows that the interpretation guarantees the semantic assignment to that node. If the label is na (for "not acceptable") the interpretation guarantees that the wff does not receive that semantic assignment. Nodes which are terminal because the Semantic Domain has shrunk are not labelled (they are ungrounded and cannot be used to establish truth value.) When a terminal node is labelled, its labelling may permit the labelling of a node which directly dominates it. All upward labelling of this sort is carried out in a natural way. A semantic tree with topmost node labelled a at some finite stage is an acceptable semantic tree, grounded in those terminal nodes which contributed to the labelling of the topmost node.

Procedure 1 may fail to produce a label for the topmost node of a semantic tree and if so, Procedure 2 is used. In Procedure 2 nodes which failed to receive a label through Procedure 1 and which have semantic wffs as first members are now labelled and the nodes below them are disregarded. A wff with first member of the form Uab and third member T, for instance, would be labelled a by the second procedure, if the first procedure had left it unlabelled and (therefore) established that it was ungrounded. A node with first

member of the form Tab and third member T would be labelled na by Procedure 2 if Procedure 1 had left it unlabelled, since the first procedure has demonstrated that Tab is ungrounded and that it cannot, therefore, be true.

The Semantics of SR. If W is the set of wffs of SR, $\langle D,V\rangle$ is an interpretation, $w \in W$, and i is an integer,

w is $\left\{\begin{matrix}\text{false}\\ \text{true}\\ \text{undetermined}\end{matrix}\right\}$ on $\langle D,V\rangle$ if and only if there is an acceptable derivation tree under $\langle D,V\rangle$ with topmost node of the form $\left\{\begin{matrix}\text{Fab,W,T,V,i}\\ \text{Tab,W,T,V,i}\\ \text{Uab,W,T,V,i}\end{matrix}\right\}$, where V(a) is w.

(Note: p if and only if q must be read here as $\sim(\sim(\neg p \vee q) \vee \sim(\neg q \vee p))$.)

This semantics assigns each wff of SR a unique truth value (or the value undetermined). The proof is as follows.

Lemma 1. Removing the topmost node n of a semantic tree labelled by Procedure 1 does not alter the labels given by Procedure 1 to the subtree(s) directly dominated by n.

Proof: Follows from inspection of Procedure 1. Labelling proceeds upward from terminal nodes. Removing the topmost node of a properly labelled tree can have no effect on the labels of the nodes it dominates.

Lemma 2. There cannot be semantic trees t_1 and t_2 such that $t_1 = t_2$ and the topmost nodes of t_1 and t_2 are labelled a and na.

Proof: Follows immediately from inspection of the labelling procedure. Identical nodes receive the same labels.

Theorem 1: Of any three semantic trees with topmost nodes of the following form, at most one is labelled a on an interpretation $\langle D,V\rangle$:

(1) $\langle$Tab,SD,T,V,i$\rangle$ (2) $\langle$Fab,SD,T,V,i$\rangle$ (3) $\langle$Uab,SD,T,V,i$\rangle$

Proof: It cannot be the case that the third node is labelled a and one of the two others is labelled a. (If so the labelling procedure has labelled the tree directly dominated by the first or second node a. But trees identical to this appear under the third node and the labelling procedure would have labelled it a by Procedure 1 and lemma 1. This would force the third node to be labelled na by Procedure 1.)

It remains to prove that there are no two acceptable trees with topmost nodes of the form of (1) and (2). Assume not. Then there are acceptable trees t_1 and t_2 with topmost nodes of the form of (1) and (2) which are labelled a at some finite stage of Procedure 1 or Procedure 2. We show that this is impossible by giving a procedure for tracing parallel paths downward on t_1 and t_2 such that the nodes traced to on each path have identical first and second members and different third members. The procedure is as follows (where a T-node, F-node, or U-node is a node with T, F, or U as third member, respectively.)

1 For a terminal node, stop tracing.

2 For a node with first member of the form $\sim p$, trace to the node directly dominated.

For a T-node and an F-node with first member of the form $\neg p$, trace to an acceptable node directly dominated by the T-node (by A2 there is one) and trace to the similar node directly dominated by the F-node. (No U-node appears on one of the trees with a T-node or an F-node on the other, for reasons advanced in the first paragraph of this proof.)

3 For a T-node and an F-node with first members of the form p∨q, trace to an acceptable node directly dominated by the T-node (there is one by A3) and trace to the similar node directly dominated by the F-node.

For a T-node and a U-node of this form, trace to an acceptable node directly dominated by the T-node and trace to the similar node directly dominated by the U-node.

For an F-node and a U-node of this form, trace to an acceptable U-node directly dominated by the U-node (there is one by A3) and trace to the similar node directly dominated by the F-node.

4 For a T-node and F-node with first members of the form (Ex)∅x, trace to an acceptable node directly dominated by the T-node and trace to the similar node directly dominated by the F-node.

For a T-node and U-node of this form, some node directly dominated by the T-node is acceptable by A4. Some node with the same first and second members and F or U as third member directly dominated by the U-node is acceptable. Trace to it.

For an F-node and a U-node of this form, trace to an acceptable U-node directly dominated by the U-node (there is one, by A4.) Trace to the similar node directly dominated by the F-node.

5 For T-nodes, F-nodes, and U-nodes with first member of the form Tab, Fab, or Uab, a must name a wff of SR which is a member of the Semantic Domain. Otherwise the nodes are terminal, but this cannot occur because the labelling conditions allow no two terminal nodes with identical first and second members and different third members to be labelled a.

If the nodes traced to are of these forms, trace to the nodes directly dominated:

⟨Tab,SD,T,V,i⟩ ⟨Tab,SD,F,V,i⟩; ⟨Fab,SD,T,V,i⟩ ⟨Fab,SD,F,V,i⟩.

There are no pairs of nodes traced to of the form

⟨Tab,SD,T,V,i⟩ ⟨Tab,SD,U,V,i⟩; ⟨Tab,SD,F,V,i⟩ ⟨Tab,SD,U,V,i⟩;

⟨Fab,SD,T,V,i⟩ ⟨Fab,SD,U,V,i⟩; ⟨Fab,SD,F,V,i⟩ ⟨Fab,SD,U,V,i⟩.

The reason is that the first member of each such pair must directly dominate a semantic tree labelled a by Procedure 1. By A5 and SR5 the same semantic tree appears under the second member of each such pair, with label na. By Lemma 2 this cannot happen.

There cannot be a pair of nodes traced to of the form ⟨Uab,SD,T,V,i⟩ and ⟨Uab,SD,F,V,i⟩ since one of the two subtrees directly dominated by the F-node is labelled a by A5. By A5 and SR5 the same subtree is labelled na under the T-node. By Lemma 2, this cannot happen.

The rules just given show how to trace a path from the topmost nodes of t_1 and t_2 downward. The paths are such that the ith member of each path has identical first and second members, a third member different from that on the path on the other tree, and label a. There are two cases to consider. Either the paths are of finite length n or less and have a terminal node as last member, or they do not terminate.

The first case is ruled out by inspection of the acceptability rules for terminal nodes. No two such nodes will be labelled a on the same interpretation.

The second case is ruled out because Procedures 1 and 2 terminate at some finite stage in the labelling process, by assumption. The labelling process labels upward from terminal nodes only. Since the tracing procedure traces only to labelled nodes, it must terminate before the nth stage is reached. (There are no labelled nodes at lower stages to trace to).

The assumption that there are such trees t_1 and t_2 is false and the theorem is proved.

Theorem 2: Every wff of SR is true, false, or undetermined.

Proof: Suppose not. Then there is a wff p with name a such that Procedures 1 and 2 do not label any tree with topmost nodes of these forms a:

(1) ⟨TaW,W,T,V,i⟩ (2) ⟨FaW,W,T,V,i⟩ (3) ⟨UaW,W,T,V,i⟩

Node (3) must be labelled na by Procedure 1. (If it received no labelling by Procedure 1, Procedure 2 would label it a.) Hence one of the two subtrees directly dominated by node (3) is labelled a by Procedure 1. But this tree is identical to that directly dominated by node (1) or node (2). Hence node (1) or node (2) is labelled a by Procedure 1, which contradicts the assumption. The theorem is proved.

5. Interesting Features of the SR Semantics

In section 3 it was demonstrated that the Liar sentence "This sentence is false" is undetermined in SR. The Deferred Liar sentences ("a_1 is true" and "a_2 is false", where a_2 names the first sentence and a_1 names the second) are also undetermined, as the following semantic tree for a_1 demonstrates:

(30) $\langle Ua_1W,W,T,V,2\rangle$

$\langle Ta_2W,W\text{-}a_1{*},T,V,2\rangle$ $\langle Fa_2W,W\text{-}a_1{*},T,V,3\rangle$

The two terminal nodes are unlabelled by Procedure 1 since a_2 names a wff in $a_1{*}$. Procedure 2 then labels the topmost node of (30) a.

The Strengthened Liar ("This sentence lacks truth value") is undetermined in SR, as the following acceptable semantic tree demonstrates (assume $V(a_1)$ is "Ua_1W"):

(31) $\langle Ua_1W,W,T,V,1\rangle$

$\langle Ta_1W,W\text{-}a_1{*},T,V,3\rangle$ $\langle Fa_1W,W\text{-}a_1{*},T,V,12\rangle$

The labelling is carried out as it was for (30).

These results appear desirable. Now let us consider some which seem at first blush to be less so.

We cannot say of the three Liar wffs we have considered (the Liar, the Deferred Liar, and the Strengthened Liar) that it is true that they are undetermined. To see this, let a_1 name any of these sentences and let a_2 name "Ua_1". Then the relevant semantic trees are these:

(32)

$\langle Ua_2W,W,T,V,3\rangle$

$\langle Ua_1W,W\text{-}a_2{*},T,V,4\rangle$

etc.

$\langle Ta_2W,W,T,V,2\rangle$

$\langle Ua_1W,W\text{-}a_2{*},T,V,3\rangle$

etc.

The topmost node of (32b) is not labelled by Procedure 1 and Procedure 2 labels it na. The topmost node of (32a) is similarly unlabelled by Procedure 1 but Procedure 2 labels it a. Thus, in the SR semantics, "The Liar is undetermined" is undetermined in truth value. The appearance theory of truth, that where a names p, p implies Ta--is not true of the SR semantics. This fact has a conse-

quence which appears quite strange: the semantics itself is undetermined! The relevant semantic tree is this (where s names the wff which formulates the semantics, the interpretation of "≡" is as before, and where $\begin{Bmatrix}\text{"TTxy"}\\\text{"UTxy"}\\\text{"FTxy"}\end{Bmatrix}$ means "There is an acceptable semantic tree with topmost node of the form $\begin{Bmatrix}\langle V(x),y,T,V,i\rangle\\\langle V(x),y,U,V,i\rangle\\\langle V(x),y,F,V,i\rangle\end{Bmatrix}$)":

(33)

n_1 $\langle UsW,W,T,V,3\rangle$

n_2 $\langle(x)(y)(Txy\equiv TTxy\&Fxy\equiv FTxy\&Uxy\equiv UTxy),W\text{-}s^*,T,V,2\rangle$

n_3 $\langle Tb_1b_2\equiv TTb_1b_2\&Fb_1b_2\equiv FTb_1b_2\&Ub_1b_2\equiv UTb_1b_2,W\text{-}s^*,T,V,3\rangle$

etc.

(For simplicity, the logical equivalent for "~(~p∨~q)" is "p&q". The node intermediate between n_2 and n_3 has been omitted. $V'(b_1)$ is the Liar and $V'(b_2)$ is W-s*.)

Node n_3 is not labelled by Procedure 1 since the left-hand side of each equivalence is not labelled by Procedure 1. Hence n_2 and n_1 are unlabelled by Procedure 1 and n_1 is labelled a by Procedure 2. The semantics rules itself undetermined.

That the theory is undetermined would be a fatal consequence if it were a Tarskian theory of truth, for according to the Tarskian theory (as Kripke explains it in Kripke (1975)) we are entitled to assert a sentence exactly in those circumstances in which it is true. The semantics of SR is non-true and, according to a theory based on Convention T, it should not be asserted.

Within a groundedness theory of truth, however, this consequence (that the theory is undetermined in truth value) is acceptable, for to be true a sentence must be grounded by Procedure 1 in wffs which

make it true. To be undetermined it must receive no truth value by Procedure 1. The semantics does not receive a truth value by Procedure 1 because it discusses the semantic assignment to wffs which do not themselves receive assignments by Procedure 1. Hence it is correct, according to a groundedness theory, that the semantics is undetermined.

However, one can (and ought to) assert the semantics for SR. In fact, the semantics is a special case of a rule for introducing lines of SR proofs:

(34) SR line introduction rule: If there is an acceptable semantic tree with topmost node of the form $\langle p,SD,T,V,i\rangle$ and sd names SD, then the following is an acceptable line of a SR proof:
sd p.

This rule allows one to assert the semantics over any Semantic Domain. The relevant semantic trees will have topmost nodes of this form:

(35) n_1 $\langle(x)(y)(Txy\equiv TTxy\&Fxy\equiv FTxy\&Uxy\equiv UTxy),W,T,V,j\rangle$

n_2 $\langle Tb_1b_2\equiv TTb_1b_2\&Fb_1b_2\equiv FTb_1b_2\&Ub_1b_2\equiv UTb_1b_2,W,T,V',k\rangle$

(one node is omitted as before, for simplicity.)

Consider n_2, the result of any instantiation to x and y. Each conjunct of n_2 has on the left a wff which dominates a semantic tree of the sort syntactically described by the wff on the right. Procedure 2 labels every node of the form of n_2 a. (The reader may verify this by writing the wffs out and working out the trees.) Thus n_1 is labelled a by Procedure 2 and the semantics is legitimately asserted.

It may appear that this rule leads to contradiction. Let us discuss this point with reference to the Universally Quantified Liar ("All sentences which have the property of being the Universally Quantified Liar are not true"). Let UQ be a predicate satisfied only by the wff $(x)(UQx\rightarrow\sim TxW)$ and let u name this wff. Then the Universally

Quantified Liar is vacuously true in SR. The relevant semantic tree is this:

(36)

n_1 $\langle TuW, W, T, 3\rangle$

n_2 $\langle (x)(UQx \rightarrow \sim TxW), W\text{-}a^*, T, 2\rangle$

etc.

Every node directly dominated by n_2 is labelled a by Procedure 1 because no allowable instantiation satisfies UQ. Hence n_1 is labelled a.

One would think that the following contradiction was derivable from this fact: (Let "(x)∅x" abbreviate "∼(Ex)∼∅x" in this discussion):

(37) TuW (assumption)

(38) (x)(UQx→∼TxW) (from 30 and Convention T)

(39) UQu→∼TuW (instantiation to u)

(40) UQu (definition of UQ)

(41) ∼TuW (from (39) and (40))

According to this proof, (37) contradicts (41). However, proofs in SR are not to be carried out with the unrestricted rules of Convention T. In particular, these deduction rules are the counterparts of the two implications of Convention T:[8]

(42) a. Where $V(y) \subset V(SD)$,

$$\frac{SD \quad Tay \qquad SD \quad V(a)=p}{y\text{-}a^* \quad p}$$

b. Where $V(a^*) \subset V(SD)$

$$\frac{SD\text{-}a^* \quad p \qquad SD\text{-}a^* \quad V(a)=p}{SD \qquad TaSD}$$

The counterpart of the disappearance theory of truth is found in (42a). In SR, when p is inferred the Semantic Domain is shrunk so that members

of a* are removed from it. The SR version of the appearance theory is found in (42b). In SR, if a names p and there is a semantic tree with topmost node of the form ⟨p,SD-a*,T,V,i⟩ which is labelled a by Procedure 1, then we may conclude TaSD. (As we have seen, not every semantic tree is labelled by Procedure 1. For those of the correct form which are so labelled, (42b) allows the inference allowed by the appearance theory.)

The fact that lines of proofs consist of the name of a Semantic Domain as well as a wff requires a reformulation of the rules of quantification. In particular, the rule of Universal Instantiation is

(43) Where Øa SD and a is a constant of SR, $\dfrac{\text{SD } (x)Øx}{\text{SD } Øa}$

Now let us examine the reasoning in (37)-(41) as it would be carried out in SR.

(44) W TuW (by line introduction)

(45) W-u* (x)(UQx→~TxW) (from (44) by (42a))

The instantiation to u cannot now be carried out, for the SR instantiation rule allows no instantiation which creates a wff not in the Semantic Domain. As we have seen, every allowable instantiation produces a wff assertable over W-u*. Hence the contradiction does not arise.

It is worth considering what advantages are gained by accepting the groundedness account of truth as opposed to Tarski's account. There are four advantages which should be mentioned. First, the groundedness theory eliminates the need for the cumbersome machinery of language hierarchies--counterintuitive machinery whose appeal is based solely on the need to avoid contradiction at any cost.

Second, the grounded ness theory gives some content to the truth predicate, whereas a theory based on Convention T does not. According to the disappearance theory of truth, the truth predicate is dispensable, having no more content than the sentence the predicate holds for. Perhaps it is to be viewed as a syntactic variant of sentence assertion which is to be employed in the interests of brevity or stylistic variation. Surely there is more to truth than this! In the groundedness theory the truth predicate does have content. It asserts of sentences over Semantic Domains that they are grounded. The notion of the groundedness and of the Semantic Domain are content which is not provided by a mere assertions of sentences themselves. Third, the semantics is formulable in the theory and is assertable in the theory. This, I believe, is the strongest argument for it. The intuition which prompted the investigation resulting in the SR semantics was that we talk about the semantics of English in English, and we are entitled to do so. In this respect, the SR semantics is preferable to a semantics based on an objectlanguage-metalanguage distinction. Fourth, everything which could be said in the Tarski language hierarchies can be said in SR, and all inference and truth values which held in the Tarski language hierarchies under Convention T hold for the SR translations of the language hierarchy wffs as well. (The proof of this statement is highly formal and not directly relevant to Bhartṛhari's work. It is found in the Appendix.)

The SR semantics uses the great insight of Tarski--that discourse about truth requires restrictions on the sentences talked about--without the drawbacks of Tarski's Solution. Furthermore, it provides

a framework in which to formalize the theories of Bhartṛhari as he intended them to be formulated. For these reasons it has been specified here in some detail.

6. The Compatibility of the Vākyapadīya and the SR Semantics

The SR semantics is not intended to be a formal representation of a theory of truth which Bhartṛhari held. Rather, it is a system within which Bhartṛhari's theories may be presented as he intended them to be, given what later investigation has determined about the dangers of semantic discourse. Let us consider some respects in which the SR semantics and the Vākyapadīya are related.

Discussion of Liar Paradoxes. Bhartṛhari gives an example which appears to be an instance of the Universally Quantified Liar in this passage:

> (46) The sentence 'all that I am saying is wrong' is not literally meant. If what it says is wrong, the point in question would not be conveyed. (VP III 3.25, Iyer)

In order to be a Liar sentence, the sentence discussed in (46) must satisfy two conditions. It must be the case that the other things the speaker has said are indeed false, and it must be the case that the sentence itself is one of the sentences under consideration. Bhartṛhari's remarks imply that the sentence does not imply quantification over itself. Pragmatically, we must remove it from the domain of the discourse because if we do not an unintended result will arise. Bhartṛhari does not say what that result is; if he had, we might have had a discussion of Liar phenomena in the Vākyapadīya.

There is no other text I find which discusses liarlike phenomena. Thus, while the solution to the paradoxes has no textual base in the Vākyapadīya, there is also no passage which weighs against it.

Truth Value Gaps. When discussing sentences in the informal way, Bhartṛhari sometimes used the definition of the Mīmāṃsakas, referring to fitness as a feature which might be taken into consideration in evaluating a sentence. As we have seen in Chapter I, some philosophers of the West have treated sequences of words which have expectancy satisfied but which lack fitness as sentences without truth value. By ascribing this theory to Bhartṛhari, one might then argue that the step to assigning no truth value to ungrounded sentences is a small one. However, there is no discussion of semantics truth value gapping in the Vākyapadīya, and I am not sympathetic to the argument that sentences which lack fitness lack truth value. So I have no basis for using such a theory as a representation of Bhartṛhari's thought. On the other hand, there seems to be no text which militates against using truth value gaps in this way either.

The Use of Metalanguages. As Tarski's Solution dominates Western semantics at present, it is reasonable that researchers attempting to reinterpret the philosophers they are studying should search for evidence that his theory was foreshadowed in the work of the Indian philosophers. This line of investigation has been pushed furthest by Hartmut Scharfe in Scharfe (1971), a work entitled Panini's Metalanguage. Scharfe describes the purpose of the book in the following way:

(47) The following pages are intended as a grammar of the metalanguage used in Pāṇini's grammar, the Aṣṭadhyāyī....It is evident at first sight that the metalanguage has been modelled after the object language (Sanskrit); even in its perfected state it has not severed all ties with the object language. (p. 5)

What Scharfe fails to do in the book is to support his claim that Pāṇini's language was a metalanguage for Sanskrit. In fact his work shows more or less clearly that what he calls a metalanguage for Sanskrit is really an abbreviated version of Sanskrit which is adequate to discuss its own syntax. This point is touched on by Scharfe himself:

(48) There are some rules, in which Panini avoids the 'shorthand' expression of the metalanguage and describes the facts instead in the manner of the object language. (p. 46)

As we have seen earlier, for one language to be a metalanguage for a second according to Tarski (the originator of the term) it must contain a name of every expression of the second and it must contain a truth-predicate. There is no indication that the language Pāṇini used to state his grammatical rules contained a truth predicate. In fact, Misra in Misra (1966) quotes Chomsky's Syntactic Structures approvingly: "I think that we are forced to conclude that grammar is autonomous and independent of meaning..." and goes on to say

(49) Pāṇini intuitionally, without the modern equipment of mathematical modelling, evolved a system of description which (1) is independent of consideration of meaning as a factor in description of forms... (p. 111)

Thus there seems to be no evidence that Pāṇini was using a metalanguage in his Aṣṭadhyāyī and, if Scharfe is right in the passage quoted, what he is doing is employing "shorthand" for ordinary Sanskrit expressions in order to compress the rules he gives.

To say that Pāṇini is using Sanskrit "shorthand" as his language is to say that Sanskrit has in it abbreviation operators such as "In this work I use X to stand for Y." Such operators cast a spell over the book so that in the book, before doing the semantics of a string of words, we must disenchant them. That is, we must substitute Y for X before going ahead with a standard semantics for Sanskrit. The fact that such operators yield standard Sanskrit sentences when disenchanted shows that their use does not put us in a language different from Sanskrit. The fact that their use is restricted to particular domains (such as "this work") may have led Indian philosophers to say that Sanskrit is a permanent, uncreated language whereas the language of the Grammarians is nonpermanent and is created by men (Scharfe, p. 2). (This view is not one which Bhartṛhari held). These philosophers (and Scharfe) are mistaken in thinking that another language is employed by the Grammarians, but their point is understandable all the same. It looks as though entirely new symbols and syntax are being used in the enchanted sentences. Caught in the spell, one is apt to think one is using a new language.

My belief is that the formal languages in which modern researchers do semantics (the language of the formalism in section 3, for example) is the result of a similar convention. When we have it told us in logic classes that a sentence of the form "p∨q" is true if and only if at least one of p and q are true, what we are learning is a shorthand device for representing ordinary English. The instructor is saying, "When you see sentences of this form written by logicians, what they mean is ..." Such statements throw an abbreviational spell over

logic books such that, were the spell to be broken, the sentences of the "formal languages" would appear in their true guise: sentences of ordinary English. All this is possible given the SR semantics, but not given a Tarskian view. Furthermore, it seems to me to be a correct description of the way I learned formal languages and the way I think about and in them. According to the theory of formal language learning just advocated, there is some mystery about the way we learn ordinary English but, given that we have learned it, there is no longer any mystery about the way we learn the languages of logic (or Pāṇinian syntax).

J.F. Staal, in an investigation of the use of metalanguage apparatus by the Indian philosophers, is more circumspect than Scharfe. He writes in Staal (1975):

(50) If there is a parallel between Indian and Western concepts, it is certainly not very apparent. The notion of 'metalanguage' occurs in Western logic almost always in the context of the analysis of truth, and refers in general to formalized languages....But the notion of truth, despite its importance in Indian civilization from the Vedas onward, has never played a very explicit role in Indian logic or linguistics, and fully formalized languages or fragments of such languages do not occur in Indian culture outside the areas of mathematics, astronomy and grammar. (pp. 315-316)

The formalized languages of grammar do not seem to be metalanguages in the Tarskian sense. The same seems to hold for the other cases, in which the formalism involves abbreviatory devices for numbers and formulae. In order to discuss parallels between the Indian and Eastern use of formalized fragments of language, Staal uses a weaker characterization of the metalanguage relation than that Tarski used:

(51) An object-language is a language consisting of expressions which refer to non-linguistic objects....A metalanguage is a language consisting of expressions which refer to the expressions of an object-language....The notion of metalanguage may be related to other notions, e.g., 'artificial language' or 'technical language'. (pp. 316-317)

In this article (called "The Concept of Metalanguage and its Indian Background") Staal discusses the phenomena noted by Scharfe and others using the definition of metalanguage given in (51). It is interesting to note that he finds no uses which fall under the Tarskian characterization of the metalanguage relation. For our purposes, then, I take it that there is no need to use the metalanguage apparatus of Tarski in representing the theories of Bhartṛhari. While the Indian philosophers used notation which metalanguages must use (names for syntactic expressions and translations of longer expressions) they did not do any work explicitly using metalanguages in the Tarskian sense. The articles which have been quoted are the result of a metamania induced by Western reliance on Tarskian solutions to Tarski's Proof. This is, in my view, an aberration of our times which is better disposed of than encouraged.

A Correspondence Semantics. The SR semantics assigns truth values depending on the way the sentence under consideration corresponds to the interpretation. If we think of the interpretation as a representation of the world, then we may say (as it is often said) that the semantics is a way of relating our language to the world.

Something like this notion is found in the following passages of the Vākyapadīya:

(51) In verbal usage, there is another Being, a secondary one, which presents the real nature of things in all circumstances (VP III 3.39, Iyer)

Helārāja's commentary on this verse is summarized by Iyer in this way:

> (52) Such a Being consists in their figuring in the mind. It is called aupacārikī, to distinguish it from Being outside the mind, in the external world. (Iyer, p. 98)

Later on in this section, Bhartṛhari writes:

> (53) No meaning of a word can go beyond this secondary Being.... which is the cause of the use of all words.
> (VP III 3.49 Iyer)

Iyer's summary of Helārāja's commentary on this passage reads:

> (54) Thus, all words move in the realm of this secondary Being.... When we use the word 'asti' [it exists] in regard to a thing, what we are doing is to say that it has outside reality in addition to having secondary Being. Even external Being becomes capable of being expressed by words only when it is grasped by the mind. (Iyer, p. 105)

Bhartṛhari goes on,

> (55) Verbal communication relates only to a part of an aspect of reality or to the determination by means of an external factor or to a reversal of reality or to an absence of it.
> (VP III 3.52, Iyer)

In these passages is found the following features of a correspondence semantics: the way we describe things need not be the way they really are; the way we use words may not result in true sentences--it is the link between speech and the outside world which determines whether or not we have spoken truly; we have the apparatus for describing relations between our speech and reality (it seems that a free logic may be being used here, in that Bhartṛhari and Helārāja seem to allow for the "mental" existence of nonexistent objects); we determine whether or not we have spoken truly by determining whether or not the relation or aspect of reality we have spoken of obtains as we have said.

The Changing Nature of Sentences from Node to Node. In the SR semantics the assignment to a wff may change depending on the Semantic Domain over which it is being evaluated. Since the Semantic Domain may change as one goes down a tree, at different nodes on the same tree a sentence may have different semantic assignments. One way to describe this feature of the semantics is to say that a sentence may change in meaning during the evaluation process--or, viewed from the standpoint of other sentences, it may alter in character. Bhartṛhari says something parallel to this in the following passage:

(56) When knowledge in the form of doubt has assumed the form of śeṣa (meant for something else), it cannot become the object of another doubt without losing its original form.

(VP III 3. 23, Iyer)

(57) What is expressive cannot at the same time be the expressed. What conveys something else cannot at the same time be conveyed by something else. (VP III 3.26, Iyer)

In (56), Bhartṛhari suggests that the character of a statement may change when it is the object of another statement. In (57) he suggests that being expressed by another expression may be the same sort of situation. This feature of his system is compatible with the SR feature that a sentence may express a second sentence which, in turn, expresses the first with a different Semantic Domain. The first has lost its original character as meta on the second and has become the object of a sentence which, at that node, is meta on it.

How the SR Semantics is Introduced into the Fragment. To add the SR semantics to the fragment, we make the following additions to the lexicon. To the constants add p_1, p_2, ..., intended as names for sentences of the fragment. To the adjectives (category CN/CN)

which already include amithyā ('true'), mithyā ('false'), and unda ('undetermined'), add satyavṛkṣa, mithyāvṛkṣa, and undavṛkṣa ('has an acceptable semantic tree with x, W, T (or F or U respectively) and V as first four members', where x is the argument of the adjective). It should be noted that these adjectives are my own invention entirely, except for mithyā. Generally, satya seems to be used when "true" would be used in English, but it is also used to mean "real", and I have used it that way in the fragment. Finally, una is added to the category of sentence modifiers. (una is the negation whose translation is "¬"--again my own invention.)

Then, where p iff q means una p vā q ca una q vā p, the semantics for the fragment may be expressed in the fragment as follows:

(58) sarvam vākyam satyam vākyam iff satyavṛkṣam vākyam tad sarvam ca vākyam mithyam vākyam iff mithyāvṛkṣam vākyam tad sarvam ca vākyam undam vākyam iff undavṛkṣam vākyam tad.

We may read (58) as "Every sentence is a true sentence iff it is a truth-treed sentence and every sentence is a false sentence iff it is a false-treed sentence and every sentence is an undetermined sentence iff it is an undetermined-treed sentence." I assume that the pragmatic factors discussed in Chapter I will rule this the most preferred of the possible readings of (58).

The mechanism for describing the syntactic predicates on which the semantics is based is not found in the fragment, because it would involve too much expansion of the fragment to too little end to add the logical machinery needed. However, the intended interpretation of the predicates is that given in the previous sections of this chapter, and it should be clear how the fragment might be enlarged to include such a specification, were the project to be undertaken.

FOOTNOTES

1 In developing the semantics described in this chapter I have relied a great deal on the comments, suggestions, and instruction of Terence Parsons. Helpful comments on earlier versions of the semantics for SR (usually in the form of telling counterexamples) were provided by Earl Conee, Edmund Gettier, Alan McMichael, Terence Parsons, James Waldo, and Rick Wiley.

2 Tarski's Proof, Convention T, and a discussion of their implications are found in Tarski's "The Concept of Truth in Formalized Languages," p. 155 of Logic, Semantics, Metamathematics, Oxford Press, 1956.

3 This position has striking similarities to the Third Man Argument in Plato's Dialogues.

4 For an exposition of an alternate formulation of the Kripke paper and a simpler version of the tree symbolism which is used in the SR semantics, the reader is referred to Davis (forthcoming), to be published in the Journal of Philosophical Logic.

5 Throughout the upcoming discussion I use SLS predicates which are assumed to be another way of writing SR predicates in standard SR form. Although "Rāma" is not a constant of the SR specifications, for example, it is understood that it is another way of writing "a_i", for some integer i.

6 It is assumed for simplicity that identity is a predicate which satisfies the usual axioms. Henceforth it is written as "=" rather than in its standard SR form.

7 The negation operator "¬" makes a true sentence false, a false sentence true, and an undetermined sentence true. This is in contrast to the negation operator "~", which makes a true sentence false, a false sentence true, and an undetermined sentence undetermined.

8 I have not completed the development of a set of rules of inference for SR. The idea is straightforward enough, but as always there are tricky details to keep watch on.

It is a straightforward matter to show that there is no acceptable semantic tree with topmost node of the form $\langle P\&\sim P, SD, T, V, i\rangle$. Proofs of the consistency of rules of inference need only show, then, that if there are acceptable semantic trees for the premises, there is an acceptable semantic tree for the conclusion. In most cases this is a simple matter, but the exceptions are not yet worked out.

CHAPTER V

GRAMMAR AND BRAHMAN

In the preceding chapters we have investigated Bhartṛhari's theory of words, a theory which was insightful but which was regarded by him as of less value than his theory of sentences. The theory of sentences was insightful, but it too was ultimately regarded by him as of less value than a third theory according to which sentences do not exist. A study of Bhartṛhari's work which fails to discuss this theory must distort his position to a great degree, for it is a continually recurring theme in the Vākyapadīya that Grammar leads one toward salvation and that, having done so, it is dispensible.

These ideas are treated most fully in Vākyapadīya, Book III, sections 1-4, where Bhartṛhari discusses the relation between grammar and Brahman, the ultimate reality in the Vedic tradition. In the process of the discussion a good many things are said which are puzzling to the careful reader. Some of them (that Brahman cannot be expressed, for example) appear contradictory and some of them (that study of Grammar leads one to salvation, for example) seem bizarre. In this chapter an interpretation of Book III is developed in which one may read Bhartṛhari's remarks in a consistent way. The formalism of the preceding chapters is used in developing this interpretation. Each section of the chapter concerns one puzzling point, together with an interpretation which renders it unproblematic.

It is not my intention in what follows to present a theory which I believe is true. (Indeed, I do not know whether the theory of Advaita Vedānta is true; moreover, as I interpret it, there is doubt whether any way of formulating it is correct.) Instead my intention is to show how the system may plausibly be interpreted so that it does not fall to the obvious objections and so that what appear to be problematic passages make sense.

1. Assertions About Brahman

Consider these passages from the Vākyapadīya and Helārāja's commentary on it:

(1) That one Reality is seen as the word, the meaning and their relation.[1] (VP III 2.14 Iyer)

(2) Brahman...is the highest universal (VP III 1.33, Helārāja as summarized by Iyer)

(3) The Reality...is beyond all assertions....Thus it is not possible to make any positive assertions about the Reality.[2] (VP III 2.12, Helārāja as summarized by Iyer)

In (1) is found the Vedic doctrine that Brahman is the only thing which is real. In (2) it is asserted that this Reality is the highest universal. In (3) it is denied that assertions may be made of it. How are we to reconcile (3) with the assertions made about Brahman in (1) and (2)?

This point is not explicitly addressed in the Vākyapadīya or its commentaries, but there are several passages which bear on it indirectly. For example:

(4) Remaining on the path of Unreality one strives after Reality. (VP II 238, Pillai)

Sometimes passages of this nature are interpreted to mean that all language is useless, or that contradiction is a part of the unreal world in which we find ourselves and cannot be avoided. Combined with (3), they are sometimes taken to mean that discourse is useless and that Reality must be sought without thought. To render Bhartṛhari's position in this way would be to do it a great injustice. If Bhartṛhari is interpreted as one who blithely espouses contradictions in order to provoke the cessation of thought, he is made out to be a poor philosopher, and he was not that. A better reading of such passages follows.

There is reference in (4) to a path one follows through Unreality. In considering the nature of this path it seems clear that (1) and (2) describe one's position at a different stage of it than that described by (3). As we shall see, Bhartṛhari was a proponent of a system of personal development called vāgyoga (speech-yoga). Although very little is now known about this system, according to Iyer it involved stages of progress analogous to three stages of speech described in Book III of the Vākyapadīya:

> (5) ...some kind of Yoga practice for the attainment of Brahman-śabdatattva is an integral part of the philosophy of Bhartṛhari who...thought of the process as a kind of ascent from the differentiated to the totally undifferentiated.
> (Iyer (1969), p. 142)

In order to interpret the passages which we consider in this chapter, it is necessary to postulate three stages of this path.

At the first stage one holds what might be called the common-sense metaphysical position. To hold such a position is to believe that there are entities in the world--cows, pots, people, etc.--and

that sentences are true or false depending on whether they describe these entities correctly. One interpretation of language based on the common-sense position might be that the sentence Rāmas dhāvati 'Rāma runs' of SLS is true if and only if the entity named by 'Rāmas' is running. As we shall see, Bhartṛhari thinks that such interpretations may be of two types. They have one important feature in common, however. On the common-sense position in either version the adjective satya 'real' holds of many men, women, cows, pots, etc.

Bhartṛhari's view was that the common-sense position is that of the philosophically unsophisticated individual. It is not the position of one who has studied the Vedic treatises on the true nature of Reality, for their subject is Brahman, often described (as in (1)) as the only real thing. Let us consider the meaning of the proper name Brahmanas 'Brahman' in the NP category of SLS. It seems that its meaning cannot be fixed for a language learner by ostension, for according to the Vedas Brahman may not be perceived. Perhaps it would be introduced by definite description in this way:

(6) Brahmanas $=_{df}$ satyas bhavin 'Brahman is the unique real being'

Suppose the noun specifier in (6) to be pragmatically determined as the definite description specifier, as in the translation of (6). Then no true positive assertions about Brahman may be made since, where P is a predicate asserted of Brahman, the translation of the assertion will be of the form

(7) $(Ex)((y)(satya(y)\rightarrow x=y)\&Px)$

No assertion of this form will be true on a common-sense interpretation because according to the common-sense position there is no unique real

entity. (More complicated positive assertions will fall to the same difficulty, as long as each constituent discusses the nature of Brahman.) This I take to be the explication of (3), that Reality is beyond all positive assertions--no positive assertion about Brahman is true.

Consider another metaphysical position, that held by someone who has undergone experiences which have convinced him or her of the following theses: (1) there exists an entity which alone is real, (2) the other entities named in ordinary speech are not real, and (3) the entity which is real may not be thought about in the way in which pots, cows, and people are thought about (we are not aware of thinking about it, nor are we conscious of it when we are thinking about it). These are beliefs of one school of Advaita Vedānta, beliefs which it appears Bhartṛhari held.

Let us represent the common-sense position by all the sentences of the SLS fragment which are true on the common-sense interpretation. Examples are

(8) a. na narasiṃhas bhavin satyas 'it is not the case that a man-lion is a real entity'

b. na Brahman bhavin satyas 'it is not the case that Brahman is a real entity'

c. Rāmas naras 'Rāma is a man'

Let us call the second theory the AV theory. A proponent of the AV theory would interpret the fragment just as the common-sense theorist did, with one difference: the predicate satya 'real' will hold only of Brahman on an AV interpretation. In the AV version of the fragment, those sentences not containing the words satya

or Brahman will have the same truth values as in the common-sense theory. Only sentences containing satya or Brahman may differ in truth value. For example, according to the AV interpretation,

(9) na brahmanas bhavin satyas 'it is not the case that Brahman is a real entity'

is a false sentence, whereas it was true on the common-sense interpretation.

There are no contradictions in (1)-(3) when we view (1) and (2) as proposed truths of the AV theory and (3) as a statement about the common-sense theory. So viewed, (3) should be read

(3') [according to the common-sense theory] reality is beyond all assertions...

What appears to be a difficulty about all language is, according to the interpretation I am advancing, a theory about properties of interpretations of Sanskrit.

I believe the following passage suggests that the interpretation just advanced is on the right track:

(10) Similarly, when forms such as earth disappear, primordial substance, that is Brahman, remains. For this, the authority is the written tradition. It would not be right to say that all this universe proceeds from something which is non-existent and inexpressible. (VP III 2.15, Helārāja's commentary as rendered by Iyer. Italics mine.)

In the context of the written tradition--the Vedas--Helārāja is willing to say that assertions may be made about Brahman. This passage comes two pages after the commentary in which Helārāja asserts (3). There the context is discussion of Brahman when one is concerned with the world of appearances--the world of the common-sense theory.

2. Are Monistic Positions Inconsistent?

There is a third stage on the path of vāgyoga which is sometimes described as that of attaining Brahman. It consists of an experience which is characterized as lacking form or content, but which is valued above all others by people in the Vedantic tradition. The position one holds at this, the final stage of the process of achieving salvation, is described by Bhartṛhari and Helārāja as Monism, the view that there is only one thing. (This may not be the best description of it, for reasons to be detailed in section 5, but it will do for now).

One difficulty with monistic positions is that it does not seem that they can be consistently expressed. To express any position is to use language--that is, to use a word or words whose meaning is what the position is. In this use of language we must have at least two things--words and meanings--as well as a person who is expressing the theory. Consequently to express a monistic position seems to be to give the grounds for denying it.

There are several solutions to this difficulty which do not render Bhartṛhari's position accurately. One is to resolve the difficulty by arguing that sentences about cow, pots, and people which are true on the AV interpretation are true in the way sentences in novels and plays are true of their characters--that is, they are not actually true, but we may speak as if they were in order to amuse or instruct ourselves. The difficulty with this solution is that the monistic position ought to be actually true if it is to be adequately rendered, and on this solution even talk about Brahman will not be

really true. At best it will be true in a fictional sense, but this is not good enough. Bhartṛhari's theory was that the third stage of the process described earlier was the highest, and yielded the ultimate insight. It will not do to interpret it as a fictional insight.

Another unacceptable solution is to argue that the Monism which Bhartṛhari held is not properly described by saying that there is only one thing. Rather, it is the theory that there is only one ultimately real thing. This view is compatible with an expression of it, if one's view is (as is the AV view) that only Brahman is ultimately real. According to this position there are unreal entities--mirages and things in dreams and stories--and there are real entities--the people, cows, pots, etc. which exist--and there is one ultimately real entity--Brahman. We may talk about all these things.

This theory is the pariṇāmavāda theory, one of the classical versions of Vedānta. According to the pariṇāmavādins the things in the universe which we perceive and discuss are transformations of Brahman--real transformations, although less real than Brahman. They are real enough, for example, to be members of a universe of discourse or to be used as referents of philosophical terms, although they are not real enough to explain the existence of all the things which we find in the world.

The pariṇāmavāda theory is often contrasted with the vivartavāda theory of Advaita Vedānta, according to which the things we perceive and talk about, as well as the perceivers and talkers, are not real. Only Brahman is real, according to the vivartavada theory.

Iyer notes that it is not clear whether Bhartṛhari was aware of

the distinction between these two theories, nor is there agreement among his commentators concerning which one he held. M. Biardeau in Biardeau (1964), p. 10, notes that there is no text in the Vākyapadīya stating directly that the things in the universe are unreal. On the other hand, Helārāja and the Indian commentators in his tradition tend to the interpretation of Bhartṛhari as a vivartavādin, as does Iyer. (For a discussion of the points of scholarship involved, the reader is referred to Iyer (1969), pp. 16-22 and pp. 128-135.) What is striking about this discussion is that Iyer admits that no passage in the Vākyapadīya directly resolves matters, and he is forced to use some complicated inferences from terminology in other texts in order to support the vivartavāda interpretation. It is curious that this should be so, for there is strong philosophical evidence that Bhartṛhari held the vivartavāda position, evidence which is clear to a critical student of the monistic position, although it is not evidence of the sort which a scholar like Iyer is used to seeking out and it is not evidence which someone steeped in the monism of Sankara is liable to notice (Sankara seems to talk in the way I have labelled "second stage" or "AV" but seems to express third-stage positions with the second-stage sentences. If one accepts this way of talking as legitimate (and non-contradictory) then one will not believe that the point about to be made is of value.)

Formally, to interpret Bhartṛhari as a pariṇāmavādin would not be difficult. The word satya 'real' would be interpreted in such a way that when modified by an adjective meaning 'ultimately', its extension would be only Brahman.

This version of the Vedānta school of philosophy has been held by many Indian thinkers, but it is clearly not the position Bhartṛhari held, for the following reason. The theory is straightforwardly expressible in the fragment of SLS interpreted in the way just described. The only problems one enounters concern making sense of the word "ultimately". According to Bhartṛhari, on the other hand, there are deep difficulties in expressing the truth about the third stage of vāgyoga. For example,

(11) That vision of the sages which is based on Reality cannot be put to ordinary use; their vision is not linked with words. (VP II 139, Pillai)

(12) ...the true knowledge which is indescribable is pointed out as having grammar as its means of realisation. (VP II 2 34, Pillai)

I see no difficulty in linking the pariṇāmavāda vision with words, nor can I ascertain any reason, on the pariṇāmavāda view, why the true knowledge should be indescribable; there are, however, such difficulties on the vivartavādin view.

I take it, then, that Bhartṛhari is correctly interpreted as a vivartavādin. Given such an interpretation, however, we are left with the problem that the theory cannot be consistently asserted.

A better solution to this difficulty is the following. For many Western philosophers it is standard working procedure to attempt a fair formulation of a position under consideration and then use that formulation as a basis for deductions of its consequences. If the system turns out to have contradictory consequences, as the monistic position does, this is regarded as a sufficient condition for rejecting it. I want to argue against such rejection that this is not a sufficient condition for rejecting a position.

Consider the sentence "X never communicates", taken to be true of a person X. If X were to express this sentence, it would be false. In fact, the sentence expresses a position which X cannot consistently express. This does not mean that X cannot consistently hold the position expressed by the sentence, for there is no inconsistency in X's believing or entertaining the proposition expressed by it as often as X likes.

The monistic position is similar to this one in the respect that, while it cannot be consistently expressed by one who holds it, it can be consistently held. (Or at least, a position much like it can be consistently held. See the remarks in section 5). Let us follow Bhartṛhari as I have interpreted him in identifying the holding of positions with the undergoing of flashes of insight. Then, on Bhartṛhari's theory, to express a position p is to give a sentence which has p as its meaning. To hold a position p is to experience the flash of insight p. (Perhaps to keep on holding it the conditions are less stringent, but this seems to be the way one begins to hold the position.)

This distinction is important in discussion of the problem of monistic inconsistency because we see that, as in the description of X's non-communicativeness, a monistic position may not be consistently expressed by one who holds it. It may, however, be consistently held. The way in which one holds it is, by Bhartṛhari's account, to experience Brahman--an experience which he describes as follows:

(13) Purity of knowledge consists in its embracing all objects and not having (sense-contact as its) basis. When no form of objects figures in it, purity, some say, reaches a still higher stage. (VP III 3.56, Iyer)

Helārāja comments on this verse,

(14) In [the knowledge of the omniscient's] finished form, it is pure Consciousness like the sea without the slightest ripple on its surface. It is the supreme Brahman. (Iyer's summary).

The highest state of vāgyoga, the state of pure consciousness in which there is no form or differentiation, is described in (13) and (14). It is compatible with the account of this state offered in these passages that one in it would not be able to describe it. The state is one in which there is nothing to describe, nor is there anyone apparent to do the describing. A well-known Vedic verse describes this state in the following way:

(15) For where there is a duality (dvaita) as it were, there one sees another;...there one understands another. Where, verily, everything has become just one's own self, then whereby and whom would one smell?...then whereby and whom would one understand? Lo, whereby would one understand the understander?[3] (Bṛhadāraṇyaka Upaniṣad, II.iv.14, Hume)

As in (11) this state is a vision which would not be linked with words for the one who has it. As in (12) it is a state which would be indescribable by the person who knew it. In this interpretation lies the explanation of what is sometimes called "The Paradox of its inscrutability":

(16) It is conceived of by him by whom It is not conceived of.
He by whom It is conceived of, knows It not.
It is not understood by those who say they understand It.
It is understood by those who say they understand It not.
(Kena Upanisad, II.3, Hume)

There is no paradox in describing this state, as long as one allows the distinction between holding a position and describing it.[4]

3. Contradictory Assertions about Brahman

Bhartṛhari wrote one verse which contains a series of contradictory statments:

(16a) It does not exist and it does; it is one and it is many; it is connected and it is separated; it is transformed and it is not. (VP III 2.13, Iyer)

There is really no need for me to interpret this passage so that it appears consistent, for Helārāja has done just that. I mention it only because there is a widespread belief among Western philosophers that many Indian philosophers openly espoused contradictions, or tacitly believed that the world is a contradictory place. To my knowledge, this belief is not founded on fact, although passages like (16a) certainly seem to support it. Iyer's summary of Helārāja's commentary on this passage is as follows:

(16b) And yet it is Brahman which appears as everything else. It appears as positive entities and as negative entities. It appears as one and as many (as one in the case of universals and as many in the case of the individuals.) It appears as associated with things and as separated from them. It appears as transformed and as not transformed. (Iyer)

In interpreting (16a) a pragmatic factor allows us to derive a sentence meaning from (16a) which is different from the primary meaning of (16a). It is more accurate (but less dramatic) to phrase the thought expressed in (16a) in this way:

(16c) Some appearances of it exist and some don't; some appearances of it are single and some many; some appearances of it are transformed and some are not.

Read in this way, there is no intended contradiction in (16a) and no contradiction is intended to be read from it.

4. Why Hold the Position?

If the position one holds at the third stage of Bhartṛhari's vāgyoga is one which cannot be consistently expressed, what possible reason would a person have to adopt it? Descriptions of it carry inside themselves the seeds of their refutation, being entities different from their hearers and, therefore, parts of a plural world. Although the monistic position may be formulated by someone who does not hold it, there is no way to present a formulation of it to a person in such a way that the person (holding a common-sense interpretation of language) could consistently accept it. This fact seems to constitute grounds for rejecting the position, but I shall argue in this section that it is not.

In describing, evaluating, and modifying our philosophical views we use languages which are interpreted. It is on the basis of such interpretations that we believe the sentence "Brahman is real" to be true, false, or (for some, perhaps) undetermined in truth value. Sometimes it is demonstrated to us that an interpretation makes certain of our beliefs inconsistent. A well-known example of such inconsistency is the problem of free will and determinism, in which it comes as a surprise to many people that their beliefs about free will, determinism, and moral responsibility entail a contradiction. In such cases we may modify our beliefs, our interpretations, both, or (if puzzled or lazy) neither. Philosophy is an endeavor in which one attempts to hold positions which are true. Because inconsistency is a hallmark of a false position, the philosophically respectable response to contradiction is to alter one's position so that one's

beliefs are compatible. A feature of the alteration process which is of special interest to us is that thought about the contradiction and description of alternative positions are carried out with the old beliefs and in the language interpreted in the old way--that is, inconsistently. It is with the aid of the defective interpretation and beliefs that one thinks out new interpretations and beliefs to hold, and it is in the terms of the old that one characterizes the new. In our struggle for truth and consistency it may happen that the very tools we use--our words--are shown to be unreliable. When this happens, we forge new tools with the old and go on, because we have no other tools to use.

This appears to be the situation with respect to Bhartṛhari's third stage. People who have undergone experiences characterized in (13) and (14) describe them in words. In so doing they cease to hold the position which they describe. The flash of insight which is without content is replaced by flashes containing words, meanings, and worldly interaction. Nonetheless the position is one which is believed by such people and in the very language in which the position is false they set about describing what it is and the way to go about holding it again. Remarks in language about Monism, then, are of the status of remarks of one who is endeavoring to escape contradiction with the aid of a language and an interpretation which he or she wishes to reject. (Perhaps Tarski's attempts to create a consistent language out of natural language is another example of this sort of endeavor.)

I take this to be the explication of Bhartṛhari's remark in (4) that, "Remaining on the path of Unreality, one strives for Reality." One uses language to go as far as one can and, having experienced a position which goes beyond it, one jettisons the language. This point is illustrated in another passage from the Vākyapadīya, reminiscent of a later remark by the philosopher Wittgenstein:

(17) (Grammarians) propound means (for the understanding of language) which, once grasped, can be thrown overboard.
(VP II 38a, Pillai)

5. Is the Position Monism?

The discussion has proceeded on the assumption that the position one holds at the third stage of vāgyoga is the position that there is only one thing, Brahman. If the experience one undergoes is as described in (13) and (14), however, it is doubtful that Monism is the proper rendering of it. According to Bhartṛhari and Helārāja, the experience is "pure consciousness", undifferentiated. A more accurate rendering of it would seem to be something like "Brahman exists", or, as described in this passage, "It is":

(18) Not by speech, not by mind,
Not by sight can He be apprehended.
How can he be comprehended
Otherwise than by one's saying 'He is'?
(Katha Upanisad 6.12 Hume)

The "only" in the monistic position that there is only one thing must have some form as its basis. Either the position held at the third stage is different from the description Bhartṛhari and Helārāja give us or it is not correctly described as Monism. The problem is, what is the source of the "only"?

Several answers suggest themselves. One is to conclude that the description in (13) and (14) is not completely accurate, and that there is a faint ripple of consciousness somewhere that there are no other things when one holds the third stage position. This solution does not seem to me to be a good one, for it introduces into the third stage position the elements which would render it inconsistent--conceptions of things other than Brahman. The position would then be subject to the internal inconsistencies discussed in section 2.

Another solution is to take the obvious way out and deny that the third stage position is monistic. It might then be called "Brahmanism" or some other name, purged of the restrictive connotations of "Monism". As described the position is after all of a thing rather than of a thing beside which there are no others. According to this solution, Monism would be a second stage position, an untenable position which is a steppingstone to the desired third stage. This solution would not be acceptable to Bhartṛhari for two reasons. First, he describes Monism as the truth rather than a step on the way to it, and second there are passages in the Upaniṣads which do the same and Bhartṛhari's aim was to achieve compatibility with the Upaniṣadic doctrines. There is a long tradition of describing the third stage position in Monistic terms, and it would not be appropriate to interpret this tradition as a mistake if there were a more reasonable interpretation available.

The solution which seems best is the following. The third stage position itself is not accurately described as Monistic, but taken together with the transition to and from a second stage position, it is so described. We have not discussed here what it is supposed

to be like to go from the second stage in which one uses conceptual apparatus which one believes to be discredited to the third stage in which the differentiated world disappears. One analogy found in the Vedas which is used by Bhartṛhari is that the process is like waking up from a dream, in that what was once thought to be real is discarded in favor of a new view of reality:

> (19) Just as, in a dream, the one mind appears in contradictory forms...in the same way, while the ultimate reality is unborn, eternal and devoid of inner sequence, we see it as having birth and other contradictory attributes.
> (VP III 2.17-18, Iyer)

Helārāja comments on this passage in this way:

> (20) One should not wonder that all the plurality which we see before us is being denied and the unity which one does not see is being advocated. Because that kind of thing is happening all the time. The world which we see in our dreams is contradicted in the wakeful state. Similarly, the world which we see in the wakeful state does not persist in the turiya (the state beyond deep sleep). So that may also be looked upon as unreal. (Iyer's summary)

According to the interpretation of these passages which I am advancing, the third stage position is not Monistic, but it is reached through a process of belief that everything one experiences and thinks about at stage one is dreamlike and unreal. The position at stage three is described by people at stage two who have been to stage three and returned. Such people characterize Monism as the truth, given that what they are talking about is not only the stage three position, but the process of getting to it and returning, in which everything which once seemed to be real fades into a single, undifferentiated state of consciousness which is perceived to be real in the process of the fading.

It has not been my practice in the body of this work to discuss the many parallels between Bhartṛhari's theory and other theories, Eastern and Western. The temptation to do so briefly at this point is overpowering, for the Monism of Bhartṛhari and the interpretation I have advanced for it are strikingly similar to the Monism of the Eleatic philosopher Parmenides as interpreted by Montgomery Furth in Furth (1968). At the conclusion of a fascinating exposition of Parmenidean elenchus, Furth writes that Parmenides' conclusion is

(21) The only true thought is the thought that it is. (p. 130)

The comparison with (18) is striking. In a postscript entitled "On Saying What Cannot Be Said", Furth argues that Parmenidean statements such as "Thou canst not be acquainted with what is not, nor indicate it in speech" which seem self-contradictory are instead to be regarded as devices to speed the process of imparting the truth:

> (22) Ideally, Parmenides should say nothing at all, but instead should administer some simple negative reinforcement--e.g., hitting Betathon over the head--each time Betathon 'says what is not'; and Betathon being assumed an apt pupil, he might be hoped in sufficient time to get the idea. (p. 131)

In this respect Furth's interpretation of Parmenides differs from mine of Bhartṛhari, since I do not believe a person who holds the Monistic position is capable of deciding on appropriate reinforcement, nor would such a person be capable of administering the reinforcement at the correct time, for to do so would involve understanding the words Betathon utters. After all, according to the theory, there is nobody to do the reinforcing and nobody to reinforce, nor are there utterances to call for the appropriate reinforcement. The theory of stages seems in this respect better than a reinforcement theory of Furth's type.

6. Grammar and Salvation

In this section the basis for Bhartṛhari's claim that the study of Grammar is an aid to salvation is discussed. There are several passages in the Vākyapadīya in which the claim is made. The most extensive occurs in Book I, 10-22. Some excerpts from this passage follow:

> (23) The wise say that grammar, nearest to the Brahman and the foremost spiritual training is the most important (of such) subsidiary texts of the Veda.
>
> It is a direct path towards that holiest of lights, that supreme essence of the kind of speech which has assumed distinctions of form.
>
> Words are the sole guide to the truths about the behaviour of objects; and there is no understanding of the truth about words without grammar....
>
> It is the first rung on the ladder towards liberation; it it the straight Royal Road for those desirous of (reaching) that goal.
>
> That pure light which is the supreme essence of speech... --that Supreme Brahman is attained by having recourse to grammar. (Pillai.)

The benefit of Grammar stressed most frequently in Book I is that of furnishing the student with an ability to read and speak correct Sanskrit. It was Bhartṛhari's belief that Sanskrit was a language uniquely fitted to convey human thought, and that the relation between the expressions of Sanskrit and their meanings was eternal and divinely fixed. I will not discuss this part of the position in detail because I do not find myself sympathetic to it. There is no evidence I know of to support a claim that any language is best fitted to represent thought. Given the theory of sphoṭa as I have interpreted it, any way of expressing the truth should be as good as another.

A second benefit of Grammar study mentioned in the Vākyapadīya is that of making the Vedas accessible to the student. I shall not discuss this topic either, as it does not bear on the issues I am interested in.

The third benefit of the study of Grammar is that it leads one to accept the Advaita Vedāntist position. How this acceptance comes about is one subject of Book III, sections 1-4, where the argument is the following: There are two Grammarian theories about the meanings of words. One, put forth by Vyādi and others, is that all words denote substances. The other, put forth by Vajapyāyana, is that all words denote universals. Bhartṛhari doesn't adopt either theory. Instead he argues for two theses: All theories of meaning known to him fall under one or the other of these two headings, and both theories imply the conclusion that all words denote Brahman. He concludes from these two facts that the study of any system of Grammar leads one to knowledge of Brahman.

The structure of Bhartṛhari's arguments that all words denote Brahman is extremely interesting. The points to which I shall draw attention are two: To represent these remarks adequately, the metalanguage formalism is not servicable, and Bhartṛhari's objective of producing a theory which is compatible with all the disciplines turns out to be different in practice from what one would suppose.

Concerning what a universal is, Bhartṛhari writes:

(24) Similarity consisting of absence of difference or the powers which are the very essence of things, these might be described as the synonyms of the universal. (VP III 1.92 Iyer)

(25) After all, the universal is nothing more than something in the individuals which causes a similarity of cognition. (VP III 1.92, Helārāja, Iyer)

Let us refer to a semantics which specifies a universal as the meaning of every word as a jāti (universal) semantics. One might wonder just how it is that a jāti semantics assigns universals as the denotation of words other than the natural candidates. Bhartṛhari considers several cases about which we might be puzzled. Concerning proper names, the theory is this:

> (26) Even proper names denote universals. The name of a person stands for that unchanging recognisable element which persists in all the changes which he undergoes. (Helārāja on VP III, 1.12)

Concerning verbs, adverbs, prepositions and adjectives, the theory is this:

> (27) Similarly, the verb denotes the universal aspect of action, present in the different moments of action and causing the same cognition and the use of the same word. The universal of the accessory (kāraka) conveyed by a verb plays a subordinate part in the cognition produced by a verb....According to this view, prepositions (upasarga) also express the universal, because they do no more than denote a peculiarity in the meaning of the verb which is a universal. A postposition is also based on the universal of a relation. Similarly, words like śukla (white) express the universals of qualities... (Helārāja on VP III 1.2, Iyer)

There is no mention of logical operators such as na 'not' or vā 'or' in this passage.

I shall not attempt a full explication of the theory here, especially since Bhartṛhari does not commit himself to it. My interest is in the use Bhartṛhari makes of it, particularly in his claim that two other theories of his time are theories which employ a jāti semantics.

The Nyāya-Vaiśeṣika semantics assigns universals as the denotation of some words, but as we have seen in the last chapter their theory of universals differed from that of the Grammarians in that

the Nyāya-Vaiśeṣikas believed that there are no universals over universals. As we have seen, Bhartṛhari argued that the Nyāya-Vaiśeṣikas were wrong in holding this view. With respect to the other categories proposed by the Nyāya-Vaiśeṣikas, Bhartṛhari argued that their members are universals also. The claim he makes upon completion of these arguments is that the Nyāya-Vaiśeṣika semantics is a jāti semantics. He does not mean by this that the Nyāya-Vaiśeṣikas assert that their semantics is a jāti semantics, because they do not. Rather he means that the semantic statements made by the Nyāya-Vaiśeṣikas may be interpreted by a jāti semanticist as though the meanings of the words assigned were universals.

Bhartṛhari's treatment of the Vijñānavādin school of Buddhism was more striking, in that the Vijñānavādins denied that universals existed. Nonetheless, Bhartṛhari made them out to be jāti semanticists. Helārāja's summary of their position is this:

> (28) According to the Vijñānavādins, this attempt to show that words like ākāśa [space] also denote the universal is futile, because, according to them, there is no such thing as the universal at all. They do not believe in the reality of the external world. They only believe in the different states of consciousness and, in them, some things figure as common properties while others appear as distinguishing features. A word denotes only this thing which figures in the consciousness and this is what is called jāti.
> (Helārāja on VP III 1.19, Iyer)

The Vijñānavādins specifically deny the existence of universals, but this does not prevent them from being classed as jāti semanticists in the following way:

(29) ...for Grammarians, meaning (artha) is just what the word conveys. The...principle enables them to explain the universal according to the Vijñānavādins. According to the latter, the universal is something which occurs in the mind when a word is heard. In other words, it is śabdartha [word-meaning] and for Grammarians also, artha means śabdartha. (Ibid, Iyer)

This solution seems to amount to this: The Vijñānavādins believe that common properties of states of consciousness do what universals (on the Nyāya-Vaiśeṣika view) do. So do Grammarians. But Grammarians call these common properties universals. Therefore the Grammarian notion of universal is employed by the Vijñānavādins. (Unlike the Nyāya-Vaiśeṣikas, Bhartṛhari does not commit himself to the existence of an external world at stage two, and he is able to assimilate the Buddhist theory, in which only the contents of consciousness exist, to his own.)

These two cases throw light on a point which puzzled me for some time. Iyer and Helārāja both point out that it was important to Bhartṛhari that his theory be "common to all the disciplines" (this goal was discussed in connection with the problem of universals over universals in the last chapter). The meaning of this statement is not that every statement of every discipline ought to be true in the Grammarian system, for the Nyāya-Vaiśeṣika assertion that there are no universals over universals and the Vijñānavāda statement that there are no universals at all are not true in Bhartṛhari's theory. The statement does not mean that the statements of the Grammarians must be compatible with those of the other disciplines, for the same reason. The claim seems instead to amount to this:

(30) The compatibility doctrine: Every true sentence of any discipline which describes grammatical speech or describes the relation of speech to meaning must be true on the Grammarian interpretation.

(There are some interesting parallels between the translation of other languages into SR and the interpretation of sentences of other systems by the Grammarians. They will not be traced here.)

Some theses of the Nyāya-Vaiśeṣika system, for example are these:

(31) a. There are no universals over universals. Actions are not universals.

b. The meaning of Rāmas dhāvati 'Rāma runs' is that the action of running inheres in the substance in which the soul named by"Rāma" inheres.

According to the compatibility doctrine as formulated in (30), it is only the statement in (31b) which must be true when interpreted in the Grammarian way. As we have seen, the interpretations of actions, substance, individuals, and relations are to be universals in a jāti semantics, so (31a) will not be true in the jāti semantics. There is a tacit distinction between those statements of the theory which are peculiarly semantic and those which are metaphysical--the distinction between the sentences of (31a) and (31b)--which underlies Bhartṛhari's theory. The theory itself concerns the way the Grammarians would interpret the statements of other schools, rather than their own interpretations. I find these two points very interesting.

Of course, if (31) is a correct interpretation of the doctrine, then the Tarskian framework would place Bhartṛhari's remarks in a language which was (at least) meta on a metalanguage for Sanskrit. But his remarks are part of the system of vāgyoga, a way to the greatest

merit. Bhartṛhari would not regard as adequate a formulation which represented those remarks in a language different from Sanskrit. (Nor would he be sympathetic to a system based on the conclusion that ordinary Sanskrit is intrinsically flawed as a vehicle for conveying truths.) The SR semantics seems a better system within which to formulate Bhartṛhari's remarks than the Tarskian system for these reasons. As already stated, it is possible to view Bhartṛhari's remarks in SR as remarks of Sanskrit which are enchanted so that they appear to be sentences of a language different from Sanskrit.

Bhartṛhari's treatment of the view that all words denote substance is similar in relevant respects and will not be considered in detail here. Those disciplines which he classes under this head--the Carvāka and Sāṃkhya--are straightforwardly substance semantics and there is little need to tinker with them to get them to fit (except that Bhartṛhari uses his definition of "substance" to put souls in the category of substance. The Sāṃkhyas place souls in a distinct category.)

Having argued that the semantic theories of his time are true in a universal semantics or a substance semantics, Bhartṛhari turns to the argument that on either theory it is Brahman which all words denote. Helārāja summarizes the argument as follows:

> (32) Jāti and dravya [substance] are only two ways of understanding Brahman. When it is thought of as the persisting feature in everything, it is called jāti. When it is thought of as a finished thing, it is called dravya. Thus both the views really stand for the same view, namely, that all words denote Brahman. (Helārāja on VP III, 1.35 Iyer)

With respect to a jāti semantics, the argument is this:

(33) It is Being which, being differentiated according to the object in which it is present, is called the universal. All words are based on that. (VP III 1.33 Iyer)

Helārāja elaborates on the nature of this Being:

(34) When words convey objects the things so conveyed have a Being distinct from their external Being. It consists in their figuring in the mind....In verbal usage, it is this secondary Being which plays the main part....When all usage can be explained in terms of this Being, if one still wants to think of some other kind of Being,...let one do so. But such a Being cannot enter into verbal usage.
(VP III 3.39, Helārāja's remarks. Iyer)

The argument goes on to assert that all words denote consciousness in the sense that they denote some aspect of it or other. There is a Vedic tradition of identifying consciousness with Brahman. Following such tradition, the conclusion is that all words denote Brahman.

It is the semantic features of this argument rather than its soundness which concern me here. With respect to the substance semantics the argument is similar. The reduction of substances to mental entities is accomplished in this way:

(35) What is called the seen, or the objective world figures in the consciousness. Its very essence is, therefore, consciousness. If it were not so, it could not be illuminated.
(Helārāja on VP III 2.14, Iyer)

The assimilation of consciousness to Brahman is carried out in this way:

(36) That which persists in all states is the only thing which is real. Consciousness is the only thing which so persists.... This experiencer, the transmigrator, being essentially conscious, is Brahman....Therefore, it is the eternal śakti [power] of Brahman which manifests the unreal world, the perceiver and the perceived and creates this world dream. It is the function of philosophers to remove this universe which is charming as long as we do not reflect.
(Helārāja on VP III 2.17-18 Iyer)

(In some ways this theory resembles a theory held by Berkeley, although its conclusion is quite different. Here, to be is to be conceived of, and the objective is to do away with conceiving.) The remarks made earlier on the unsuitability of a Tarskian semantics for rendering the jāti semantics seem to apply to this part of the theory as well. It seems to me that Bhartṛhari is theorizing about all human language in these passages, although his points are made with reference to Sanskrit. If the wider application is intended, no Tarskian rendering of the remarks will work, for the metalanguage will be in a human language and the intent is that it is included in the quantificational domain of the theory.

In all this is the basis for Bhartṛhari's belief that Grammar is the "Royal Road" to salvation. The Vedas state that salvation is realizing the nature of Brahman. The study of Grammar, according to the arguments just quoted, leads one to see that the meanings of words are mental entities and that the study of meaning is the study of consciousness--Brahman itself. No matter what theory of meaning one held in India at the time Bhartṛhari wrote, Bhartṛhari was able to describe a way to produce a grammar compatible with it which referred all meaning ultimately to Brahman. In the sense that Grammar used this way could focus all thought about anything toward thought of Brahman, it was a way to salvation. As we have seen, it was a way which was ultimately dispensed with, but it was considered useful early on.

7. Grammar as Metaphor

Discussion of the relation between word meanings and sentence meanings is a persistent theme in Books II and III of the Vākyapadīya. Helārāja encapsulates Bhartṛhari's views on that relation in a passage summarized in this way:

> (37) ...in the Science of Grammar, meanings of words, agreeing with worldly usage, are isolated for the purpose of explaining the formation of words. The sentence is indivisible and so is its meaning....So there are no word-meanings. There cannot be any question of their previous separate existence. The hearer does not understand the meaning of the whole sentence all at once. He understands it little by little, part by part and then joins the parts together....As the indivisible sentence-meaning cannot be understood in a flash all at once, the unreal word-meanings are abstracted in the middle as mere means to an end. Once the sentence-meaning is understood, they disappear. (Helaraja on VP III 4.1-2, Iyer)

This view of the relative reality of word meanings and sentence meanings was peculiar to the Grammarian school of Indian philosophy. Consider the following aspects of the theory: It is apparent to common sense that there are words and word-meanings. We communicate with words as a matter of course and when we characterize our thoughts and experiences we nearly always do it with words. Words are the linguistic entities which are the most familiar to us. We see them, hear them, and think with them as a matter of course. Sentences, on the other hand, are not so familiar. When we see a sentence we tend to see it as made up of words. When we think about sentences, we think in thoughts which are read off in words. When we introspect about sentences to determine what it is to grasp their meanings we are aware of very little, and what we are aware of we describe with words. In all this there is tremendous support for the view that words

the basic unit of thought and communication, and the sentence and sentence meaning are built up out of words and word-meanings.

Bhartṛhari spends most of Book II of the Vākyapadīya arguing against this view. It must have been an astonishing insight for the Grammarians to discover that sentence meanings do not have parts--that in understanding them words and word-meanings play no part in what is understood and it is as if they never existed. Such an insight must have been especially striking in that it turns the common sense view of language completely around. The entities which common sense takes to be most real do not exist in the mind when thought have finally been communicated, and the thoughts are only artificially divided into words when communication is initiated.

This theory has striking parallels with the vivartavādin view of the way one comes to understand Brahman. On a common sense view there are no objects more real than the things we perceive with our senses. Reflection on our perceptions and on the Vedas may lead students to undergo experiences of Brahman (the sort described earlier as stage three experiences). When that happens, the world of differentiation appears as unreal, an artificial manifestation of a higher, unified truth.

It must have struck the Grammarians that their ability to argue convincingly against the common sense view of language as made up of parts strengthens the vivartavādin position that the common sense view of reality as made up of parts is false.

Nowhere does Bhartṛhari write that these parallels make Grammar an apt metaphor for the process of getting to stage three, but it is

impossible to avoid surmising that he viewed this feature of Grammar as making it worthy of study. Perhaps the discipline of vāgyoga used the Vākyapadīya as a text for meditation. These connections between the insights of Grammar and the insights of the Vedas seem to merit further research.

FOOTNOTES

1 This passage is suggestive of the via negativa of Maimonides and Aquinas, whereby no positive assertions may be made about God, but negative attributions may be made to him. I do not believe one can read the via negativa into Bhartrhari's theory because there are many positive assertions made about Brahman--(2), for example.

2 Compare Martin Buber's statement that "God is the Being that... may properly only be addressed, not expressed." (I and Thou, Charles Scribner's sons, 1958, pp. 80-81.) Buber refers to the Upanisads in this work. Did he intend a parallel to this Upanisadic doctrine?

3 Compare Paul Tillich's statements in Systematic Theology (University of Chicago Press,,vol. I, 1951, pp. 172-3):

> If God is brought into the subject-object structure of being, he ceases to be the God who is really God....Theology must always remember that in speaking of God it makes an object of that which precedes the subject-object structure and that, therefore, it must include in its speaking of God the acknowledgment that it cannot make God an object.

Some difficulties similar to those atributed to the Monistic position and its expressibility seem to apply to Tillich's, if God really does "precede the subject-object structure."

4 There is a distinction used in this solution between holding a position and believing it. Holding a position is a stronger act than believing it, since according to the interpretation I am advancing one may believe the Monistic position at stage two while not

holding it. (I believe that holding implies believing, but I am not certain.) Bhartrhari says very little about the flashes of insight I identify with position-holding, and I am not clear in my own mind what an adequate characterization of them would say. Hence this feature of the solution is somewhat skimpy.

BIBLIOGRAPHY

1. The following texts of the Vākyapadīya are quoted in the text:

Iyer, K.A. Subramania (1971), The Vākyapadīya of Bhartṛhari, Ch. III, pt. i, Deccan College at Poona.

Pillai, K. Raghavan (1971), The Vākyapadīya, Critical Text of Cantos I and II, Motilal.

(The following texts were used in preparing the dissertation)

Biardeau, Madeleine (1964), Vākyapadīya Brahmakaṇda avec la Vṛtti de Harivṛsabha, traduction, introduction, et nots, Boccaud, Paris.

Sharma, Peri Sarveswara (1972), The Kālasamuddeśa of Bhartṛhari's Vākyapadīya, together with Helārāja's commentary.

2. Commentators on the Grammarians:

(The following commentaries are quoted extensively in the dissertation)

Brough, J. (1951), "Theories of General Linguistics in the Sanskrit Grammarians," in Staal (1972), pp. 402-414.

Iyer, K.A. Subramania (1969), Bhartṛhari--a Study of his work in the light of the ancient commentaries, Poona.

Kunjunni Raja, K. (1963), Indian Theories of Meaning, Madras, Adyar Library and Research Center.

Staal, J.F. (ed) (1972), A Reader on the Sanskrit Grammarians, Cambridge, Mass.

_____ (1969), "Sanskrit Philosophy of Language," Current Trends in Linguistics, vol. 5, pp. 499-531.

(The following commentators were useful in preparing this dissertation)

Bhattacharya, Bishnupada (1962), A Study in Language and Meaning: a Critical Examination of some Aspects of Indian Semantics, Calcutta Progressive Publishers, Calcutta.

Burrow, T. (1955), The Sanskrit Language, Faber and Faber, London.

Chakravarti, Prabhatchandra (1930), The Philosophy of Sanskrit Grammar, University of Calcutta, Calcutta.

_____ (1933), The Linguistic Speculations of the Hindus, University of Calcutta, Calcutta.

Iyer, K.A. Subramania (1972), "The Point of View of the Sanskrit Grammarians," in Staal (1972).

Misra, Vidyaniwas (1966), The Descriptive Technique of Pāṇini, an Introduction, Mouton, The Hague.

Scharfe, Hartmut (1971), Panini's Metalanguage, Philadelphia.

3. On Sanskrit Grammar:

(The following sources provide data upon which the fragment of Sanskrit treated in Chapter II is based)

Apte, Vaman Shivaram (1957), The Practical Sanskrit-English Dictionary, Poona.

Monier-Williams, Sir Monier (1899), A Sanskrit-English Dictionary, the Clarendon Press, Oxford.

Perry, Edward Delavan (1936), A Sanskrit Primer, Columbia University Press, New York.

Staal, J.F. (1967), Word Order in Sanskrit and Universal Grammar, Dordrecht, Holland.

Whitney, William Dwight (1879), A Sanskrit Grammar, Oxford University Press, Oxford.

4. Concerning Montague Grammar and its Treatment of the Fragment:

Cooper, Robin and Parsons, Terence (1976), "Montague Grammar, Generative Semantics and Interpretive Semantics," in Partee (1976).

Partee, Barbara Hall (1975), "Montague Grammar and Transformational Grammar," Linguistic Inquiry, 6.2, pp. 203-300.

_____ (1976), Montague Grammar, Academic Press, New York.

Thomason, Richmond (1972), "A Semantic Theory of Sortal Incorrectness," Journal of Philosophical Logic, 1, pp. 209-258.

_____ (ed)(1974), Formal Philosophy: Selected Papers of Richard Montague, Yale University Press, New Haven (especially "The Proper Treatment of Quantification in Ordinary English").

Waldo, James (forthcoming), "A PTQ Grammar for Sortal Incorrectness," in Marianne Mitheun (ed), Proceedings of the Albany Montague Grammar Conference, University of Texas Press, Austin.

5. Concerning the Semantic Paradoxes:

Davis, Lawrence (forthcoming), "An Alternate Formulation of Kripke's Theory of Truth," Journal of Philosophical Logic.

Kripke, Saul (1975), "A Theory of Truth," Journal of Philosophy, pp. 690-716.

Staal, J.F. (1975), "The Concept of Metalanguage and its Indian Background," Journal of Indian Philosophy 3, pp. 315-354.

Tarski, Alfred (1956), Logic, Semantics, Metamathematics, Clarendon Press, Oxford.

6. Vedic Quotations:

(The Vedic quotations come from these two sources)

Hume, Robert Ernest (1931), The Thirteen Principal Upanishads, Oxford University Press, Oxford.

Radhakrishnan, Sir Sarvepalli and Moore, Charles A., (eds)(1957), A Sourcebook in Indian Philosophy, Princeton University Press, Princeton.

APPENDIX

The wffs of any language hierarchy H are translated into SR by following this procedure:

Begin with each language L_j which is meta on no other language of H. Let the interpretation of L_j be $\langle D_j, V_j \rangle$. Let the interpretation of SR be $\langle D, V \rangle$. The translation function τ on the expressions of L_j is defined as follows:

(1) for each constant of the form a_i, $\tau(a_i)=a_{i,j}$. $V(\tau(a_{i,j})=V_j(a_i)$ when a_i names a member of D_j which is not a wff. If a_i names a wff of L_j then $V(\tau(a_i))=\tau(V_j(a_i))$.

(2) for each nonsemantic predicate of the form P^n_m, $\tau(P^n_m)=P^n_{m,j}$ and $V(\tau(P^n_{m,j})=\tau(V(P^n_m))$, where $\tau(V(P^n_m))$ is the set of n-tuples which is $V(P^n_m)$ with any wff of L_j replaced by its translation into SR.

(3) for each wff w of the form $P^n_m a_1 \ldots a_n$, $\tau(w)=\tau(P^n_m)\tau(a_1)\ldots\tau(a_n)$.

(4) for each wff w of the form p, $\tau(w)=\sim\tau(p)$.

(5) for each wff s of the form (pvq), $\tau(w)=(\tau(p)v\tau(q))$.

(6) for each wff w of the form $(Ex)(\emptyset x)$, $\tau(w)=(Ex)\sim(x\in D_i v\sim\tau(\emptyset x))$.

Now translate the wffs of those languages whose domains of interpretation include wffs of the languages just translated using clauses (1)-(6) and this clause:

(7) for each wff w of the form Ta_i, $\tau(w)=Ta_{i,j}\tau(D_j)$.

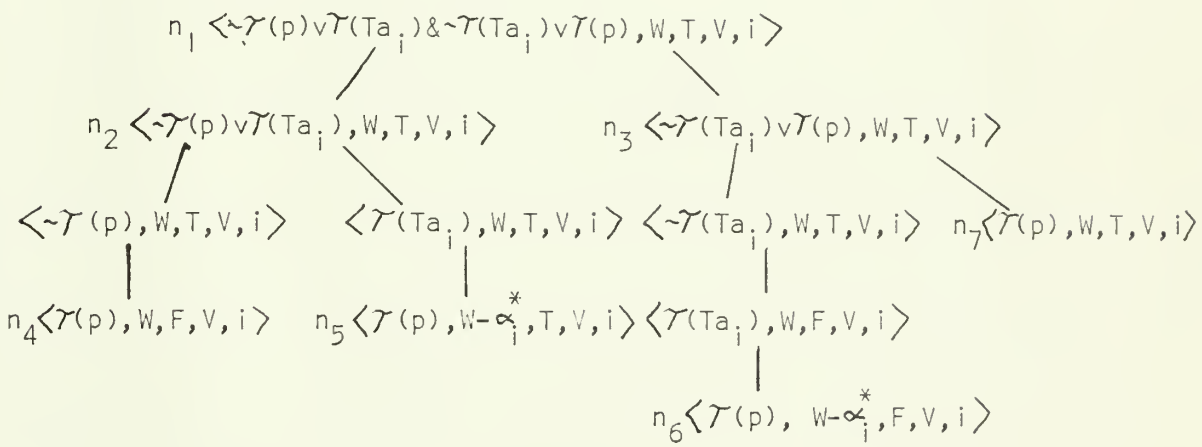
(Where D_j is the domain of L_j, (D_j) is the set of translations of the members of D_j which are wffs, together with the non-wffs of D_j as they were in D_j.)

Now we prove that, where p is a wff of some L_i of H, it is true in SR that, where a_i names p,

(8) $\mathcal{T}(p) \equiv \mathcal{T}(Ta_i)$

We may assert (8) over the Semantic Domain W if there is an acceptable semantic tree with topmost node of the form $\langle \mathcal{T}(p)=\mathcal{T}(Ta_i), W, T, V, i \rangle$. The upper nodes of such a tree will look like this ("p≡q" here abbreviates "~pvq&~qvp"):

(9)



etc.

We must show that n_4 or n_5 is labelled a and that n_6 or n_7 is labelled a. First we prove a lemma.

Lemma: Nodes n4-n7 are labelled by Procedure I.

Proof: First we show that any node of the subtrees directly dominated by n4-n7 has a translation of a wff of H as first member or is labelled immediately by Procedure I. This holds for n4-n7, for consider the nodes directly dominated by n, any of n4-n7.

If n is of the form $\langle \sim p, SD, T \text{ or } F, i \rangle$ then the node directly dominated by n has a translation of a wff of H as first member.

It cannot be that n is of the form $\langle \neg p, SD, T \text{ or } F, i \rangle$ because the condtions on $\tilde{\tau}$ yield no wffs with "$\neg$" or "U" as translations of wffs of H.

If n is of the form $\langle p \vee q, SD, T \text{ or } F, i \rangle$ then the nodes directly dominated by n have translations of wffs of H as first members.

If n is of the form $\langle (Ex) \sim (x \tilde{\xi} \tilde{\tau}(D_i) \vee \sim \tilde{\tau} \emptyset x), SD, T, i \rangle$ then the nodes directly dominated by n are of two sorts. Either the instantiation is to a member of $\tilde{\tau}(D_i)$ (and hence to a wff which is the translation of a wff of H) or it is to a non-member of $\tilde{\tau}(D_i)$. If to a non-member, the node is labelled na in two more steps by Procedure I.

If n is of the form $\langle (Ex) \sim (\sim x \tilde{\xi} \tilde{\tau}(D_i) \vee \sim \tilde{\tau} \emptyset x), SD, F, i \rangle$ then the nodes directly dominated by n are of two sorts. Either the instantiation is to a member of $\tilde{\tau}(D_i)$ or it is to a non-member of $\tilde{\tau}(D_i)$. If it is to a non-member, it is labelled a in two more steps by Procedure I.

If n is of the form $\langle \tau a_{ij} \tilde{\tau}(Dj), SD, T \text{ or } F, i \rangle$ then n directly dominates a node of the form $\langle p, SD\text{-}a_{ij}^*, T \text{ or } F, i \rangle$. From condition (1) on $\tilde{\tau}$, p is the translation of a_i in L_j of H.

These considerations apply to any node on the subtrees dominated by n4-n7. If the first member of the node is not a translation of a

wff of H, the node is labelled in two steps. Otherwise the labelling is carries out with nodes the first member of which is the translation of a wff of H.

Now we show that every dominance path from n4-n7 contains a node labelled by Procedure I. We have just seen that dominance paths which trace through nodes with first members which are not translations of wffs of H are labelled by Procedure I, so here we need only consider dominance paths containing nodes with first members which are translations of wffs of H. We see by inspection of the conditions on semantic trees that each node with a nonsemantic wff as first member directly dominates nodes with wffs which are less complex as first member. For example, a node with a wff with five logical operators as first member directly dominates a node with four logical operators as first member, and so on. Hence a dominance path from any node n contains an atomic node in k steps, where k is the number of logical operators in the first member of n. The first member of n in the dominance paths we are considering is the translation of some wff w of a language L_j of H. The translation procedure guarantees that its atomic constituents are also the translations of wffs of L_i or are of the form $x\varepsilon\widehat{\tau}(D_j)$. There are two possibilities. The atomic wff may be nonsemantic. If so, it is labelled by Procedure I. If it is semantic it is of the form $\langle Ta_{ij}\widehat{\tau}(D_j),SD,T \text{ or } F,i\rangle$. This node may be terminal because a_{ij} doesn't name a wff, in which case it is labelled by Procedure I. It cannot be the case that this node is terminal because $\widehat{\tau}(D_j)$ is not a member of SD, for suppose not. Then SD is $W\text{-}a_{i_1j_1}{}^{*}\text{-}a_{i_2j_2}{}^{*}\text{-}\ldots\text{-}a_{i_kj_k}{}^{*}$, where the $a_{i_ij_i}$ are the constants in the first member of each semantic node on the path to n. Thus there is

a member d of $\tau(D_j)$ which is a member of one of the $a_{i_1 j_i}$*. Let whichever set d is a member of be a_{mn}*. We know from the recursive definition of a_{mn}* that there is a sequence of members of a_{mn}* $w_1 \ldots w_n$ such that (1) each w_i is an atomic semantic wff, (2) w_1=d or w_1 is an atomic constituent of d, (3) Each w_i contains a name of a wff which has w_{i+1} as atomic constituent, and (4) w is $Ta_{mn}\tau(D_n)$, $Fa_{mn}\tau(D_n)$ or $Ua_{mn}\tau(D_n)$.

The definition of τ guarantees that, where d is a wff of L_d, d contains as atomic semantic constituents translations of wffs of L_d. Hence w_1 is $Tb_{k_d}\tau(D_d)$. By condition I on τ, b_{k_d} names the translation of the wff b_k names in L_d. Hence w_2 is an atomic semantic constituent of the translation of a wff of H. and w_2 must be the translation of a wff of H. Similarly for each of the w_i. Furthermore, each w_i is the translation of a wff of H which occurs in a different language of H, each of which is meta on the next by condition (Ia) on M. Thus there is a sequence of languages of H $L_d \ldots L_n$ such that d is the translation of a wff of L_d, $Ta_{mn}\tau(D_n)$ is the translation of a wff of L_n, and each language is meta on the next.

Now consider the nodes containing semantic atomic wffs on the path after the node which contains $Ta_{mn}\tau(D_n)$ as first member. Each first member of such nodes is the translation of a wff of H. Each such wff is a member of a different language of H. By condition (Ia) on M, each such language is meta on the next. Combining this sequence with the one just derived, we have a sequence of languages $L_d \ldots L_n \ldots L_j$ such that d is the translation of a wff of L_d, $Ta_{ij}\tau(D_j)$ is the translation of a wff of L_j, and such that each member of the sequence is meta on the next. But condition (Ia) on M requires that L_j be meta on

every language containing a member of D_j. Since d is a member of $\tau(D_j)$, d is the translation of a wff in D_j. Condition (1a) on M requires that L_j is meta on L_d. But this fact, together with the fact that there is a sequence of languages as just described, violates condition (1d) on M. Hence the assumption is false and $\tau(D_j) \not\in SD$.

It cannot be the case that this node is terminal because $V(a_{ij}) \not\in SD$, for parallel reasons. (Such an assumption would entail that there is a sequence of languages of H beginning with L_j and ending with L_j such that each is meta on the next. Condition (1d) on M prohibits this.)

If the node is terminal, then it is labelled by Procedure 1. If it is nonterminal the next node of the dominance path is of the form $\langle p, SD\text{-}a_{ij}^*, T \text{ or } F, i\rangle$ where A_{ij} names p. p is the translation of a wff of H and the considerations just given apply to it as well--where n is the number of logical operators in p, in n steps or less any dominance path will contain a node labelled by Procedure 1 or an atomic node. If the atomic node is terminal it is labelled by Procedure 1. If not, the next node is the translation of a wff of some language L of H. And so on.

It cannot be the case that some dominance path of this sort goes on infinitely without containing a node labelled by Procedure 1, for consider: all nodes containing as first members wffs which are not translations of members of H are labelled in two steps by Procedure 1. Those nodes containing as first members translations of wffs of H occur in the following patterns: where n is the first node containing a translation of a wff w of L_j and k is the number of logical

operators in w, the next k nodes on the path contain as first members translations of wffs of L_j. The kth node must be semantic if the path is to continue. The next node contains as first member the translation of the wff named by the constant in the kth node. By condition (1a) on M, this wff is a member of a language L_m such that L_k is meta on L_m. Since there is no infinite chain of languages of H each of which is meta on the next there will be no infinite series of translations of wffs of the languages of H in any dominance path. Thus every dominance path containing nodes with first members which are translations of wffs of H contains a terminal node of the form which is labelled by Procedure I.

Consider the subtrees directly dominated by n4-n7. Every dominance path contains a node labelled by Procedure I at some finite length from n. Let k be the length of the longest such dominance path. Procedure I labels any node which directly dominates a set of nodes each member of which is labelled. Hence in upward labelling from level k, each node of the subtree in question will be labelled by Procedure I. The Lemma is proved.

Consider the subtrees directly dominated by n4-n7. It is impossible that both n4 and n7 are labelled <u>na</u>. If so, we could trace a path downward to terminal nodes such that the members of the path on both trees have identical first and second members, one has F and one has T as third member, and both have the same acceptability labelling. (The path is traced in a fashion similar to that used in Chapter IV.)We see by inspection of the labelling rules that it is impossible to have two terminal nodes with identical first and second members, opposite third members, and same label. Hence n4 or n7 is

labelled a. (It can't be that the two nodes aren't labelled, by the Lemma). However, if n4 is labelled a so is n6 and if n7 is labelled a so is n5. We show this for n4. The removal of a_{ij}^* from W does not prevent n6 from being labelled a if n4 is labelled a, for consider. The only cases in which the difference in Semantic Domain makes a difference are the cases in which p is of the form $(Ex)\vee(\sim x\in\widetilde{\tau}(Di)\vee\sim\widetilde{\emptyset}x)$ or p is of the form $Ta_{mn}\widetilde{\tau}(D_n)$. In the first case the diminution of the Semantic Domain will make no difference, for the members of a_{ij}^* are not members of $\widetilde{\tau}(D_i)$. (If they were, we would be able to show that the languages in H failed to satisfy condition (Id) on M as in the proof of the Lemma.) In the second case the diminution of the Semantic Domain will make no difference, since $\widetilde{\tau}(D_n)$ is a subset of $W-a_{ij}^*$ (or else an analogous contradiction would be derivable.) This reasoning applies to each node of the trees.

In a similar way, if n7 is labelled a then n5 is labelled a. Hence we know that one of n4 and n5 is labelled a and one of n6 and n7 is labelled a. This is sufficient to label n a by upward labelling. The tree is an acceptable semantic tree. We may assert (8) over W.